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Optimality of cycle time and inventory decisions in a two echelon inventory system with exponential price dependent demand under credit period

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Abstract. This work describes and proposes an two echelon inventory system under supply chain, where the manufacturer offers credit period to the retailer with exponential price dependent demand. The model is framed as demand is expressed as exponential function of retailer's unit selling price. Mathematical model is framed to demonstrate the optimality of cycle time, retailer replenishment quantity, number of shipments, and total relevant cost of the supply chain. The major objective of the paper is to provide trade credit concept from the manufacturer to the retailer with exponential price dependent demand. The retailer would like to delay the payments of the manufacturer. At the first stage retailer and manufacturer expressions are expressed with the functions of ordering cost, carrying cost, transportation cost. In second stage combining of the manufacturer and retailer expressions are expressed. A MATLAB program is written to derive the optimality of cycle time, retailer replenishment quantity, number of shipments, and total relevant cost of the supply chain. From the optimality criteria derived managerial insights can be made. From the research findings, it is evident that the total cost of the supply chain is decreased with the increase in credit period under exponential price dependent demand. To analyse the influence of the model parameters, parametric analysis is also done by taking with help of numerical example.

1.Introduction

The environmental factors have led to an enlarged emphasis on supply chain management over the past several decades; they have also been important drivers in the increasing importance and varying role of the purchasing function within organisations. Globalization of business has been increased and the resulting, competitive pressures have created from a seller's to a buyer. As a result, the difficulty of buying decisions has increased and the role of the purchasing organisation in managing the buyer-seller interface has become more critical. New product development activities have been improved, due to increased rate of technological change and introduction of new product have become more resulted in greater demands on suppliers in support of their customers. The company's operations are the control of current assets being procured or produced with certain inventory management policies. Hence coordination will come in picture between two players. In this paper the coordination mechanism are introduced by manufacturer to the retailer. The coordination mechanism can be considered as credit period under the demand function as expressed in terms of exponential price dependent. This research work is presented as follows. Review of literature of the problem is carried out in Section 2. Section 3 explains development of the mathematical model comprises of the features and assumptions, notations used. In Section 4, a numerical investigation is presented. Finally, Section 5 is narrating the conclusions of the present work and indicating future scope of the work.

2.Literature Review

Many of the authors' have demonstrated different approaches for supply chain coordination. Saha et al (2015) modeled supply chain coordination contracts with inventory level and retail price dependent demand. They classified contracts as joint rebate contract, wholesale price discount contract and cost sharing contract under stock and price induced demand for two echelon supply chain coordination perspective. Du et al (2013) explored coordination of two-echelon supply chains using wholesale price discount and credit option. Chakraborty et al (2015) proposed multi-item integrated supply chain model for deteriorating items with stock dependent demand under fuzzy random and bifuzzy environments. The authors considered the non linear function as production set up cost, which is expressed in terms of production rate, suppliers transportations cost, retailer procurement cost. Under the fuzzy random and bifuzzy environment models are framed, to evaluate optimized values of credit periods and total supply chain time with the constraints of budget and space. Jaikumar et al (1990) designed a distribution systems requires two essential decisions, one is number of levels between the customer and the producer and the other is trade discounts in channel management policies. Sana (2011) proposed a dynamical system for an EOQ model for salesmen's initiatives, stock and price sensitive demand of similar products. The author considered the demand rate of product is dependent on salesmen's initiatives and selling price where display space is limited. Also, the rate of replenishment depends on the level of stock of the items. Rahdar et al (2014) proposed coordination mechanism for a deteriorating item in a two-level supply chain system. The author considered the model as several buyers and a manufacturer and assumed the inventory level decreases. Jindala et al (2016) invented integrated supply chain inventory model with quality improvement involving controllable lead time and backorder price discount. The author assumed that lead time follows normal distribution. The total cost is minimized by continuous review by taking number of shipment, order quantity, reorder point, lead time, process quality and back order price quality. Jaggi et al (2016) proposed effects of inflation and time value of money on an inventory system with deteriorating items and partially backlogged shortages. Xiao et al (2010) proposed coordination of a supply chain with consumer return under demand uncertainty. The author is design to find the profitability of the player by using buyback/markdown money contract to coordinate the supply chain under partial refund policy. Sana et al (2014) proposed a three layer supply chain model with multiple suppliers, manufacturers and retailers for multiple items. Recently the impact of future price increase on ordering policies for deteriorating items under quadratic demand are proposed by Shah et al (2016). The author proposed two scenarios about the model as 1. Special order time coincides with retailers' replenishment time and 2. Special order time falls during the retailer's sales period.In this work, two-echelon supply chain is modelled under exponential price dependent demand environment. This research attempt aimed at evaluating the optimal values of decision variables like cycle time, retailers' replenishment quantity, shipment frequencies and the total relevant cost of the supply chain with trade credit.

3. Mathematical Model Development

In the proposed model development, a two-echelon inventory system, comprising of a single manufacturer supplying a single kind of product to a single retailer is considered.

Notation

The following notations are used in the development of the model.

λ	Annual demand rate of the retailer (units/year) $(\because \lambda = ae^{-bP_R})$ where a>0, b>0 & a>>b
E_R	Retailer ordering cost (in Rs./order)
ρ_R	Fixed transportation cost of the retailer for receiving a shipment from the manufacturer (in Rs./shipment)
R_R	Unit cost at the retailer (in Rs./unit)
P_R	Unit selling price at the retailer (in Rs./unit)
W_e	Interest rate earned (in Rs./ Re./year)
W_P	Interest rate paid (in Rs./Re./year)
t or j	Permissible credit period
E_m	Fixed production setup cost (in Rs./batch)
M_m	Unit cost at the manufacturer (in Rs./unit)
ρ_m	Fixed transportation cost of the manufacturer for shipping a shipment quantity to retailer (in Rs./shipment)
q	Shipment quantity from manufacturer to the retailer in each shipment (in units)
δ	Number of shipments from manufacturer to the retailer
T or D	Length of the cycle time at retailer (in years)
φ_R	Annual total relevant cost of the retailer (in Rs.)
φ_m	Annual total relevant cost of the manufacturer (in Rs.)
φ_S	Annual total relevant cost of the supply chain (in Rs.)

3.1. Features and assumption

In the mathematical model formulation, the following features and assumptions are considered.

a) Demand rate is exponential price dependent
b) Infinite Production Rate
c) Instantaneous replenishment rate
d) Manufacturer's inventory level is integer multiple of retailer's inventory level
e) Shortages are not permitted
f) Manufacturer provides the trade credit to the retailer

3.2. Model Formulation

For the model formulation, it is assumed that the manufacturer provides the trade credit to the retailer. Inventory associated cost factors like ordering cost, carrying cost and transportation costs incurred at the retailer and manufacturer are incorporated in the model development as shown below.

3.2.1. Retailer Point

Case I: ($D \geq j$)

Annual replenishment cost of the retailer $= \dfrac{E_R}{D}$

Annual holding cost of the retailer $= \dfrac{ae^{-bP_R} DR_R W_p}{2}$ $(\because \lambda = ae^{-bP_R})$

Annual transportation cost of the retailer $= \dfrac{\rho_R}{D}$

Interest paid by the retailer per cycle $= \dfrac{R_R ae^{-bP_R}(D-j)^2 W_p}{2}$

Interest paid by the retailer per year $= \dfrac{R_R ae^{-bP_R}(D-j)^2 W_p}{2D}$

Interest earned by the retailer per year $= \dfrac{P_R ae^{-bP_R} j^2 W_e}{2D}$

Annual total relevant cost of the retailer is expressed by subtracting the interest earned per year from the sum of annual ordering cost, holding cost, transportation cost and Interest paid per year.

$$\phi_R(D) = \frac{E_R}{D} + \frac{ae^{-bP_R} DR_R W_p}{2} + \frac{\rho_R}{D} + \frac{R_R ae^{-bP_R}(D-j)^2 W_p}{2D} - \frac{P_R ae^{-bP_R} j^2 W_e}{2D} \quad (1)$$

Case II: (j>D)

Interest earned by the retailer per year $= P_R ae^{-bP_R} W_e \left(j - \dfrac{D}{2} \right)$

Annual total relevant cost of the retailer is obtained by subtracting the interest earned per year from the sum of annual ordering cost, holding cost, transportation cost and Interest paid per year.

$$\phi_R(D) = \frac{E_R}{D} + \frac{\rho_R}{D} + \frac{ae^{-bP_R} DR_R W_p}{2} - P_R ae^{-bP_R} W_e \left(j - \frac{D}{2} \right) \quad (2)$$

3.2.2. Manufacturer Point

Annual replenishment cost of the manufacturer $= \dfrac{E_m}{\delta D}$ $(\because Q_m = \delta q_R)$

Annual holding cost of the manufacturer $= \dfrac{(\delta - 1) ae^{-bP_R} DM_m W_p}{2}$

Annual transportation cost of the manufacturer $= \dfrac{\rho_m}{D}$

Annual total relevant cost of the manufacturer is obtained as the sum of annual ordering cost, holding cost, transportation cost.

$$\phi_m(\delta, D) = \frac{E_m}{\delta D} + \frac{\rho_m}{D} + \frac{(\delta - 1) ae^{-bP_R} DM_m W_p}{2} \quad (3)$$

3.2.3. Total Supply Chain

If the manufacturer and retailer are agreed to follow the joint optimal inventory decision making policy, the expression for the total relevant cost of the entire supply chain is obtained by adding the expressions representing annual total relevant cost of the retailer and manufacturer.

Case I: $(D \geq j)$

The annual total relevant cost of the total supply chain is expressed as:

$$\phi_S(\delta, D) = \frac{1}{D}\left(E_R + \rho_R + \frac{E_m}{\delta} + \rho_m\right) + \frac{ae^{-bP_R} DW_p}{2}\left(R_R + (\delta - 1)M_m\right) + \frac{ae^{-bP_R}(D-j)^2 R_R W_p}{2D} - \frac{P_R ae^{-bP_R} j^2 W_e}{2D} \quad (4)$$

Optimality Criterion:

For the given value of shipment frequency (δ), the expression representing the annual total relevant cost of the supply chain is convex in terms of optimal cycle time (D). The optimal cycle time is obtained by taking the first order and second-order partial derivatives of equation (4) with respect to cycle time. By equating the first-order derivative to zero, the optimal cycle time is obtained. Hence, $\frac{\partial}{\partial D}\left(\phi_S(\delta, D)\right) = 0$

$$\left\{2\left(E_R + \rho_R + \frac{E_m}{\delta} + \rho_m\right) + ae^{-bP_R} j^2\left(R_R W_P - P_R W_e\right)\right\} = D^2 ae^{-bP_R} W_p\left(2R_R + (\delta - 1)M_m\right) \quad (5)$$

$$\frac{\partial^2}{\partial D^2}\left(\phi_S(\delta, D)\right) = \frac{2}{D^3}\left(E_R + \rho_R + \frac{E_m}{\delta} + \rho_m\right) + ae^{-bP_R}\left(\frac{j}{D^3}\right)\left(R_R W_p - P_R W_e\right) \quad (6)$$

As the second order partial derivative, $\frac{\partial^2}{\partial D^2}\left(\phi_S(\delta, D)\right) > 0$ for all values cycle time and shipment frequency and other model parameters, the cycle time becomes optimal. From equation (5), the optimal cycle time is obtained as

$$D^* = \left(\frac{2\left(E_R + \rho_R + \frac{E_m}{\delta} + \rho_m\right) + ae^{-bP_R} j^2\left(R_R W_P - P_R W_e\right)}{ae^{-bP_R} W_p\left(2R_R + (\delta - 1)M_m\right)}\right)^{0.5} \quad (7)$$

Similarly, for the given value of cycle time (D), the expression representing the annual total relevant cost of the supply chain is convex in terms of shipment frequency (δ). The optimal value of shipment frequency satisfies the following two-inequality conditions.

$$\phi_S\left(\delta^*\right) \le \phi_S\left(\delta^* - 1\right) \quad and \quad \phi_S\left(\delta^*\right) \le \phi_S\left(\delta^* + 1\right)$$

Upon substitution of relevant values in equation (4) for the condition $\phi_S\left(\delta^*\right) \le \phi_S\left(\delta^* - 1\right)$

$$\frac{1}{D}\left(E_R + \rho_R + \frac{E_m}{\delta} + \rho_m\right) + \frac{ae^{-bP_R} DW_p}{2}\left(R_R + (\delta - 1)M_m\right) + \frac{ae^{-bP_R}(D-j)^2 R_R W_p}{2D} - \frac{P_R ae^{-bP_R} j^2 W_e}{2D} \le$$

$$\frac{1}{D}\left(E_R + \rho_R + \frac{E_m}{\delta - 1} + \rho_m\right) + \frac{ae^{-bP_R} DW_p}{2}\left(R_R + (\delta - 2)M_m\right) + \frac{ae^{-bP_R}(D-j)^2 R_R W_p}{2D} - \frac{P_R ae^{-bP_R} j^2 W_e}{2D}$$

After simplification and rearranging the terms, the following inequality is obtained as

$$\delta^*\left(\delta^* - 1\right) \le \frac{2E_m}{ae^{-bP_R} D^2 M_m W_P} \quad (8)$$

Similarly, upon substitution of relevant values in equation (4) for the condition $\phi_S\left(\delta^*\right) \le \phi_S\left(\delta^* + 1\right)$

$$\frac{1}{D}\left(E_R + \rho_R + \frac{E_m}{\delta} + \rho_m\right) + \frac{ae^{-bP_R} DW_p}{2}\left(R_R + (\delta - 1)M_m\right) + \frac{ae^{-bP_R}(D-j)^2 R_R W_p}{2D} - \frac{P_R ae^{-bP_R} j^2 W_e}{2D} \le$$

$$\frac{1}{D}\left(E_R + \rho_R + \frac{E_m}{\delta + 1} + \rho_m\right) + \frac{ae^{-bP_R} DW_p}{2}\left(R_R + (\delta)M_m\right) + \frac{ae^{-bP_R}(D-j)^2 R_R W_p}{2D} - \frac{P_R ae^{-bP_R} j^2 W_e}{2D}$$

After simplification and rearranging the terms, the following inequality is obtained as

$$\frac{2E_m}{ae^{-bP_R}D^2M_mW_P} \le \delta^*\left(\delta^*+1\right) \quad (9)$$

Combining equations (8) and (9), the inequality satisfying optimality for shipment frequency is obtained as $\delta^*\left(\delta^*-1\right) \le \frac{2E_m}{ae^{-bP_R}D^2M_mW_P} \le \delta^*\left(\delta^*+1\right)$ (10)

Case II: (D<j)

The annual total relevant cost of the total supply chain is expressed as:

$$\phi_S(\delta, D) = \frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(E_R+(\delta-1)M_m\right)-P_Rae^{-bP_R}W_e\left(j-\frac{D}{2}\right) \quad (11)$$

For the given value of shipment frequency (δ), the expression representing the annual total relevant cost of the supply chain is convex in terms of optimal cycle time (D). The optimal cycle time is obtained by taking the first order and second-order partial derivatives of equation (11) with respect to cycle time. By equating the first-order derivative to zero, the optimal cycle time is obtained. Hence, $\frac{\partial}{\partial D}\left(\phi_S(\delta, D)\right)=0$

$$2\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)=D^2ae^{-bP_R}\left\{W_P\left(R_R+(\delta-1)M_m\right)+P_RW_e\right\} \quad (12)$$

$$\frac{\partial^2}{\partial D^2}\left(\phi_S(\delta, D)\right)=\frac{2}{D^3}\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)$$

As the second-order partial derivative, $\frac{\partial^2}{\partial D^2}\left(\phi_S(\delta, D)\right)>0$ for all values cycle time, shipment frequency and other model parameters, the cycle time becomes optimal. From equation (12), the optimal cycle time is obtained as, $D^* = \left(\frac{2\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)}{ae^{-bP_R}\left\{W_P\left(R_R+(\delta-1)M_m\right)+P_RW_e\right\}}\right)^{0.5}$ (13)

For given value of cycle time (D), the expression representing the annual total relevant cost of the supply chain is convex in terms of shipment frequency (δ). The optimal value of shipment frequency satisfies the following two inequality conditions.

$$\phi_S\left(\delta^*\right) \le \phi_S\left(\delta^*-1\right) \quad and \quad \phi_S\left(\delta^*\right) \le \phi_S\left(\delta^*+1\right)$$

Upon substitution of relevant values in equation (11) for the condition $\phi_S\left(\delta^*\right) \le \phi_S\left(\delta^*-1\right)$

$$\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta-1)M_m\right)-P_Rae^{-bP_R}W_e\left(j-\frac{D}{2}\right)\le$$

$$\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta-1}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta-2)M_m\right)-P_Rae^{-bP_R}W_e\left(j-\frac{D}{2}\right)$$

After simplification and rearranging the terms, the following inequality is obtained as

$$\delta^*\left(\delta^*-1\right) \le \frac{2E_m}{ae^{-bP_R}D^2M_mW_P} \quad (14)$$

Similarly, upon substitution of relevant values in equation (11) for the condition $\phi_S\left(\delta^*\right) \le \phi_S\left(\delta^*+1\right)$

$$\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta-1)M_m\right)-P_Rae^{-bP_R}W_e\left(j-\frac{D}{2}\right)\le$$
$$\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta+1}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta)M_m\right)-P_Rae^{-bP_R}W_e\left(j-\frac{D}{2}\right)$$

After simplification and rearranging the terms, the following inequality is obtained as

$$\frac{2E_m}{ae^{-bP_R}D^2M_mW_P}\le\delta^*\left(\delta^*+1\right)\text{(15)}$$

Combining equations (14) and (15), the inequality satisfying optimality for shipment frequency is expressed as

$$\delta^*\left(\delta^*-1\right)\le\frac{2E_m}{ae^{-bP_R}D^2M_mW_P}\le\delta^*\left(\delta^*+1\right)$$

Case III: (j = D)

When the optimal cycle time becomes equal to the credit period, the annual total relevant cost of the total supply chain is obtained as:

$$\phi_S\left(\delta,D\right)=\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta-1)M_m\right)-\left(\frac{P_Rae^{-bP_R}DW_e}{2}\right)\text{(16)}$$

For given value of cycle time (D), the expression representing the annual total relevant cost of the supply chain is convex in terms of shipment frequency (δ). The optimal value of shipment frequency satisfies the following two inequality conditions.

$$\phi_S\left(\delta^*\right)\le\phi_S\left(\delta^*-1\right)\quad and\quad\phi_S\left(\delta^*\right)\le\phi_S\left(\delta^*+1\right)$$

Upon substitution of relevant values in equation (16) for the condition $\phi_S\left(\delta^*\right)\le\phi_S\left(\delta^*-1\right)$

$$\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta-1)M_m\right)-\left(\frac{P_Rae^{-bP_R}DW_e}{2}\right)\le$$
$$\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta-1}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta-2)M_m\right)-\left(\frac{P_Rae^{-bP_R}DW_e}{2}\right)$$

After simplification and rearranging the terms, the following inequality is obtained as

$$\delta^*\left(\delta^*-1\right)\le\frac{2E_m}{ae^{-bP_R}D^2M_mW_P}\text{(17)}$$

Similarly, upon substitution of relevant values in equation (16) for the condition $\phi_S\left(\delta^*\right)\le\phi_S\left(\delta^*+1\right)$

$$\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta-1)M_m\right)-\left(\frac{P_Rae^{-bP_R}DW_e}{2}\right)\le$$
$$\frac{1}{D}\left(E_R+\rho_R+\frac{E_m}{\delta+1}+\rho_m\right)+\frac{ae^{-bP_R}DW_p}{2}\left(R_R+(\delta)M_m\right)-\left(\frac{P_Rae^{-bP_R}DW_e}{2}\right)$$

After simplification and rearranging the terms, the following inequality is obtained as

$$\frac{2E_m}{ae^{-bP_R}D^2M_mW_P}\le\delta^*\left(\delta^*+1\right)\text{(18)}$$

Combining equations (17) and (18), the inequality satisfying optimality for shipment frequency, is expressed as

$$\delta^*\left(\delta^*-1\right)\le\frac{2E_m}{ae^{-bP_R}D^2M_mW_P}\le\delta^*\left(\delta^*+1\right)$$

4. Numerical Investigation

In the present section, the optimality of ordering policies and rate of shipment has been tested for a supply chain with the help of numerical data. Based on the following data, the numerical example is devised here to illustrate the model. To solve the model MATLAB program is written. Based on the step-by-step procedure developed above, the optimal values of decision variables and objective function are computed for two-echelon inventory model and the results are tabulated in Table 1.The inventory parameter values: a=6000, b=0.00009, e=0.009,
E_m = Rs. 2000/setup, E_R=Rs. 500/order, M_m=Rs. 100 per unit, R_R= Rs. 210 per unit, PR=Rs.240 per unit, λ = a*e^(-b*PR) units per year, W_P = 0.15 Rs./Re./year, W_e = 0.09 Rs./Re./year.

Table 1: Optimal Values of Decision Variables and Objective Function

Description	t = 10	t = 20	t = 30	t = 40	t = 50	t = 60
T^* (in Years)	0.0599	0.0619	0.1051	0.1051	0.1051	0.1051
q^* (in Units)	397.9	411.4	698.4	698.4	698.4	698.4
δ^* (Integer)	3.0	3.0	2.0	2.0	2.0	2.0
Q^* (in Units)	1193.9	1234.2	1396.7	1396.7	1396.7	1396.7
Φ_R^* (in Rs.)	19665.7	15082.9	13718.6	9733.0	5747.5	1761.9
Φ_m^* (in Rs.)	29615.5	29044.9	21883.2	21883.2	21883.2	21883.2
Φ_S^* (in Rs.)	49281.2	44127.8	35601.8	31616.2	27630.7	23645.1

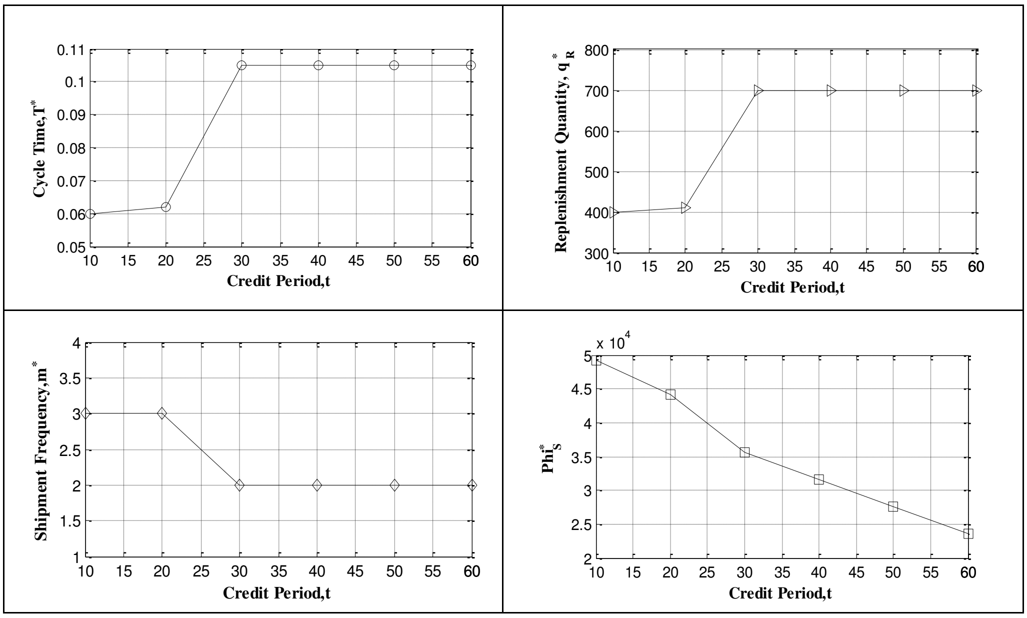

Figure. 1 Variation of Decision Variables and Objective Function
w.r.t Credit Period

5. Conclusions

The values of optimal cycle time increases with increase in trade credit. By increasing the trade credit value beyond certain period, there is an sudden increase in optimal cycle time and remain constant. The optimal retailer's replenishment quantity is more, if manufacturer permits more number of days to delay the payments of the retailer. As trade credit increasing it is shown that shipment frequencies are decreases. The model analysis describes that, the total annual relevant cost of the supply chain is less for more trade credit values. From the Figure 2 sensitivity analysis is observed that total annual relevant cost of the supply chain increases with increase in ordering cost, manufacturer set up cost and unit cost at retailer and manufacturer. The retailer's replenishment quantity increases with increase in ordering cost and set up cost and decreases with increase in unit cost at retailer and manufacturer. Similarly, the total annual relevant cost of the supply chain decreases with increase in retailers selling price. A quantitative model for two echelon inventory system is proposed for optimal values of total annual cost of the supply chain, retailer's replenishment quantity, shipment frequencies and cycle time under trade credit with the influence of exponential price dependent demand. The model describes about the study of decision variables and objective function for a supply chain under trade credit with demand is expressed as function of exponential price dependent. A MATLAB program is written to derive the optimality of cycle time, retailer replenishment quantity, number of shipments, and total relevant cost of the supply chain. Also, the parametric analysis is done to observe the behavioral pattern of decision variables and objective function with respect to variation in model parameters.

Consumer goods industries are preferred for the novelty of the present model and research findings. Managerial decisions like cycle time, retailer's replenishment quantities, number of shipments and total annual cost of the supply chain can be decided with the help of this model. The present research work can be extended to a three- echelon supply chain with wide variations and assumptions in demand function.

Table 2: Sensitivity Analysis

		t=20					t=50					t=80				
		T^*	q^*	δ^*	Q^*	TVCs	T^*	q^*	δ^*	Q^*	TVCs	T^*	q^*	δ^*	Q^*	TVCs
E_m	-40%	0.06	372.1	4.0	1488.3	54009.6	0.11	752.2	2.0	1504.3	35279.8	0.11	752.2	2.0	1504.3	27308.7
	-20%	0.05	363.7	4.0	1454.8	52644.9	0.11	725.8	2.0	1451.5	33482.0	0.11	725.8	2.0	1451.5	25510.8
	+20%	0.06	385.8	3.0	1157.5	47572.8	0.10	669.8	2.0	1339.7	29674.2	0.10	669.8	2.0	1339.7	21703.1
	+40%	0.06	428.0	2.0	855.9	43866.8	0.10	640.1	2.0	1280.1	27645.8	0.10	640.1	2.0	1280.1	19674.7
E_R	-40%	0.06	415.5	3.0	1246.6	51748.7	0.11	725.8	2.0	1451.5	33482.0	0.11	725.8	2.0	1451.5	25510.8
	-20%	0.06	406.8	3.0	1220.5	50528.4	0.11	712.2	2.0	1424.4	32558.1	0.11	712.2	2.0	1424.4	24586.9
	+20%	0.06	388.9	3.0	1166.7	48005.0	0.10	684.3	2.0	1368.5	30655.4	0.10	684.3	2.0	1368.5	22684.2
	+40%	0.05	337.4	4.0	1349.6	48345.2	0.10	669.8	2.0	1339.7	29674.2	0.10	669.8	2.0	1339.7	21703.1
M_m	-40%	0.06	374.5	3.0	1123.6	52727.8	0.10	669.5	2.0	1339.0	33667.1	0.10	669.5	2.0	1339.0	25696.0
	-20%	0.06	385.7	3.0	1157.2	51030.6	0.10	683.5	2.0	1366.9	32652.5	0.10	683.5	2.0	1366.9	24681.3
	+20%	0.06	370.9	4.0	1483.8	48818.5	0.11	714.3	2.0	1428.5	30556.9	0.11	714.3	2.0	1428.5	22585.7
	+40%	0.06	389.0	4.0	1556.2	46276.1	0.10	636.3	3.0	1908.8	29297.0	0.10	636.3	3.0	1908.8	21325.8
R_R	-40%	0.05	324.8	4.0	1299.2	55577.1	0.10	641.5	2.0	1283.1	35829.3	0.10	641.5	2.0	1283.1	27858.2
	-20%	0.05	338.7	4.0	1354.9	53486.0	0.10	668.1	2.0	1336.3	33767.4	0.10	668.1	2.0	1336.3	25796.3
	+20%	0.06	424.9	3.0	1274.7	46448.0	0.11	733.1	2.0	1466.2	29363.0	0.11	733.1	2.0	1466.2	21391.9
	+40%	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P_R	-40%	0.06	402.1	3.0	1206.3	49981.5	0.10	671.4	2.0	1342.8	28276.9	0.10	671.4	2.0	1342.8	16653.8
	-20%	0.06	400.1	3.0	1200.2	49632.4	0.10	684.1	2.0	1368.3	30021.3	0.10	684.1	2.0	1368.3	20259.3
	+20%	0.06	395.8	3.0	1187.3	48928.1	0.11	714.3	2.0	1428.6	33062.5	0.11	714.3	2.0	1428.6	26814.0
	+40%	0.06	393.5	3.0	1180.5	48573.2	0.11	732.3	2.0	1464.7	34360.2	0.11	732.3	2.0	1464.7	29768.3
W_P	-40%	0.05	341.2	3.0	1023.6	57378.5	0.09	618.9	2.0	1237.9	37719.4	0.09	618.9	2.0	1237.9	29748.2
	-20%	0.06	365.9	3.0	1097.8	53528.6	0.10	655.1	2.0	1310.1	34759.7	0.10	655.1	2.0	1310.1	26788.6
	+20%	0.07	441.7	3.0	1325.1	44504.4	0.11	751.6	2.0	1503.1	28249.7	0.11	751.6	2.0	1503.1	20278.5
	+40%	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
W_e	-40%	0.06	394.0	3.0	1181.9	49093.0	0.10	657.9	2.0	1315.8	28166.2	0.10	657.9	2.0	1315.8	17006.6
	-20%	0.06	396.0	3.0	1187.9	49186.9	0.10	677.2	2.0	1354.4	29913.0	0.10	677.2	2.0	1354.4	20347.7
	+20%	0.06	400.0	3.0	1199.9	49375.9	0.11	721.6	2.0	1443.3	33271.5	0.11	721.6	2.0	1443.3	26894.6
	+40%	0.06	401.9	3.0	1205.8	49471.0	0.11	747.4	2.0	1494.8	34873.9	0.11	747.4	2.0	1494.8	30091.2
a	-40%	0.05	473.0	3.0	1419.1	57153.0	0.09	826.3	2.0	1652.6	33952.8	0.09	826.3	2.0	1652.6	22793.2
	-20%	0.05	437.0	3.0	1310.9	53424.6	0.10	765.0	2.0	1530.0	32967.0	0.10	765.0	2.0	1530.0	23401.6
	+20%	0.07	355.1	3.0	1065.4	44588.5	0.12	624.6	2.0	1249.3	29783.8	0.12	624.6	2.0	1249.3	23406.9
	+40%	0.08	306.9	3.0	920.6	39120.9	0.14	540.9	2.0	1081.9	27273.3	0.14	540.9	2.0	1081.9	22490.6
b	-40%	0.06	406.3	3.0	1219.0	50181.5	0.10	712.7	2.0	1425.4	31931.7	0.10	712.7	2.0	1425.4	23629.5
	-20%	0.06	402.1	3.0	1206.4	49729.6	0.10	705.5	2.0	1411.0	31774.9	0.10	705.5	2.0	1411.0	23639.8
	+20%	0.06	393.9	3.0	1181.6	48836.2	0.11	691.3	2.0	1382.6	31455.9	0.11	691.3	2.0	1382.6	23645.3
	+40%	0.06	389.8	3.0	1169.3	48394.7	0.11	684.3	2.0	1368.6	31294.0	0.11	684.3	2.0	1368.6	23640.8

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