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# Optimality of Inventory Decisions and Shipment Policies in a Two-Echelon Inventory System under Quadratic Price Dependent Demand

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**Abstract**

This document is proposing an inventory system that is two-echelon for a supply chain that is coordinated and non-coordinated with one retailer and manufacturer. The retailer experiences the demand that is dependent on quadratic price and assumed to be price driven. Profit is completely dependent on the effectiveness of supply chain. Profit is the difference between the gross revenue and the total cost, whereas total cost includes carrying cost, transportation cost along with the cost of annual ordering. For a coordinated and non-coordinated supply chain, a mathematical model is being introduced. The primary objective of the mathematical model is to show the effective ordering quantity of the retailer, effective lot size of the producer, and the optimal quantity of the transportation from manufacturer to the retailer. The impact of the model parameters on objective function and variables is demonstrated with the help of a sensitivity analysis.


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*Keywords:* Two-echelon Inventory system; Quadratic Price dependent demand; Annual net revenue; Supply chain

## 1. Introduction

The supply chain includes retailer, distributor, producer, supplier along with consumer. In general these consumers will be having different objective in their mind, which are contradicting among them. Every part of the supply chain has her own inducement and information state, and nobody has the ability to upgrade whole supply chain execution. In a two-echelon supply chain under this situation, both retailer and the manufacturer need to amplify their overall revenues leading to twofold marginalization of the framework. This issue could be wiped out, just, when whole

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supply chain is worked in a facilitated manner. Along these lines, in a supply chain that is two-echelon synchronization among retailer and manufacturer is a vital issue to enhance the channel execution.

Due to the coordination of the supply chain, the downstream associate will receive proper contract parameters from upstream associate so that their benefit augmenting targets are adjusted to the goal of the supply chain. An appropriately outlined coordination contract totally wipes out twofold marginalization, as well as attains win–win circumstance for every stakeholder of the supply chain.

One of the major areas that contribute towards productivity of the manufacturer is the technology being used in day to day process. There will be a considerable development in productivity if there is an effective design with respect to inventory management, which includes planning and controlling the inventory. Effective inventory management will help to cut the cost drastically. Banerjee[1] has introduced an innovative and effective concept of Joint Economic Lot Size (JELS). The basic assumptions of Banerjee are that continuous production will be carried out by the manufacturer as per the single retailer order. In general, these retailers experience the deterministic demand. The value chain partners are the retailer and manufacturer. Manufacturer is equipped with complete information of the product demand from end customer and they will have great association with retailers which in turn will help manufacturer to get the knowledge about the customer. Many more writers like Miller and Kelle [5], Hall [2], Agrawal and Raju [4], and Li *et al.* [3], have also demonstrated that the combined thoughtfulness of economic lot size substantially minimizes the total combined cost per annum.

In this study, we attempted and proposed a two-echelon inventory system for the optimality of inventory decisions and shipment policies under quadratic price dependent demand for a non-coordinated and coordinated supply chain. We considered a model supply chain where one product is being supplied to a single retailer by a manufacturer in a quadratic price dependent demand. The structure of the work in brief is, a comprehensive literature review is discussed in Sec. 2, section 3 is a discussion of two model problems, and section 4 consists of detailed explanation of operational performance of coordinated and non-coordinated system with the help of an illustrative example. At the end, in section 5 there are a few concluding remarks along with the future scope of the work.

## 2. Literature Review

In most recent three decades, broad examination has been performed in the zone of coordination of supply chain.

Scientists have investigated different parts of organizing supply chain under a varied set of presumptions. Under established price-conscious demand these issues are analysed. One among the significant issues in supply chain administration is to discover a suitable system to facilitate supply chain to upgrade the aggregate framework execution. In the literature different systems were proposed, that includes buy back option, discount on quantity, credit facility, flexible quantity contract. Quantity discount contract is considered as the most effective and popular among the available mechanisms due its ease (Cachon, [6]).

Monahan [7] analyzed a vender implementing lot-for-lot production with infinite production rate and the merchant offered quantity discount to its client to expand order quantity. By including merchant's cost of inventory carrying Monahan's [7] model was generalized by Benerjee [8]. In addition to that Lee and Rosenblatt [9] have released the assumption that the merchant functions on a lot-for-lot system and produced more results that are generic. The models that are based on the static demand in response to quantity discount are Wang and Wu [11], Weng and Wong [10]. By taking multiple buyers into consideration they have taken the earlier studies forward. In addition, most of the studies have considered demand that is price-sensitive, for case in point, Weng [14],Weng [13], Parlar and Wang [12], Viswanathan and Wang [15]. We referred to Sarmah *et al.* [17], and Munson and Rosenblatt [16] for a broad study of the research on quantity discount.

Enormous amount of attention was paid towards inventory system several decades in the past. Wide ranging study of concerned research is available in Song *et al.* [20], Goyal and Giri [19], and Nahmias [18]. On the other hand, good number of scholars described the best possible inventory system from one company's point of view and totally ignored the optimization of the cost or profit of the complete supply chain. Supply chain synchronization has turned into an imperative segment for upgrading the productivity and responsiveness of the chain. At the point where there is no coordination, the supply chain parts act autonomously to augment their own particular benefit, which does not guarantee that the parties all in all achieve an ideal result (Sajadieh and Jokar,[21]) both from financial and ecological perspectives. At the point where there is coordination, the aggregate store network benefit/expense is expanded/minimized. However, the benefits out of coordination will go to the merchant as the purchaser is the person who will be operation off its ideal approach. Notwithstanding, the losing party is generally repaid in facilitated supply chain (Jaber and Zolfaghari, [22]). This work explores the inventory strategy for a merchant-purchaser supply chain when the players have private (benefit) and societal (natural) goals to accomplish.

Accomplishing compelling coordination among a supplier and its buyer(s) is one of the most important managerial concern and a difficult research area (Qin et al. [23]). The merchant-purchaser coordination issue has been accepting an expanding consideration by specialists and academicians (Jaber and Zolfaghari [22] and Toptal et al. [24]). In this work, the stream of study is addressing the purchaser-merchant coordination issue is alluded to as a joint economic lot sizing (JELS) issue, with the latest survey in Glock [2].

An EOQ model that was proposed by Wahab et al. [26] for the composed supply chain in two-level, considering blemished things and ecological effect, that is consolidated into the ideal strategy of the supply chain considering carbon discharge costs. Also, a two-level supply chain scientific model incorporating emanation expenses identified with producer's methods. According to the European Union Emissions Trading System (EU-ETS) tax and emission penalty was taken into account by them, and crated good number of numerical illustrations in order to demonstrate the most advantageous resolution approach over varied likely situations.

A mathematical model was introduced by Darwish and Goyal [28]by considering the single-buyer single-vendor problem under vendor managed inventory configuration. The other related recent articles addressing supply chain coordination mechanism under price driven demand include, Kim et al. [29] and Parthasarathi et al. [30]. Nagaraju et al. [31] developed a two-echelon inventory system with the price sensitive demand considering the effect of wholesale price index (WPI) and consumer price index (CPI). Syam sundar et al.[32] introduced a two level supply chain system under linear price driven demand for the optimality of replenishment quantity, inventory ratio and gross profit of the supply chain.

**Nomenclature**

| | |
|---|---|
| D | Annual demand rate of the retailer (units/year) $=\alpha-\beta.P_R-\gamma.P_R^2$ where $\gamma>0$, $\beta>\gamma$, $\alpha>>\beta$ |
| $P_R$ | Unit selling price at the retailer, in INR/unit |
| $S_R$ | Retailer's ordering cost per order for the cycle time, T (in INR/order) (INR = Indian Rupee) |
| $S_m$ | Manufacturer's setup cost per setup for the cycle time, $\lambda T$ (in INR/setup) |
| $C_R$ | Retailer's unit cost (in INR/unit) |
| $C_m$ | Manufacturer's unit cost (in INR/unit) |
| $\tau_R$ | Fixed transportation cost of the retailer for receiving a shipment quantity from the manufacturer (in INR/shipment) |
| $\tau_m$ | Fixed transportation cost of the manufacturer for shipping a shipment quantity to retailer (in INR/shipment) |
| $Q_R$ | Shipment quantity in each shipment from the manufacturer to replenish the inventory at the retailer for the cycle time, T (decision variable) ($Q_R$=DT) (in units) |
| $\lambda$ | Number of shipments from the manufacturer to retailer for the cycle time, T(integer, decision variable) |
| $Q_m$ | Replenishment batch size at the manufacturer to replenish the inventory at the retailer for the cycle time, $\lambda$T ($Q_m=\lambda Q_R=D(\lambda T)$) (in units) |
| k | Interest rate, (in INR/INR/year) |
| $\psi_R(Q_R)$ | Annual net revenue of the retailer expressed in terms of $Q_R$ |
| $\psi_m(\lambda, Q_R)$ | Annual net revenue of the manufacturer expressed in terms of $\lambda$, $Q_R$ |
| $\psi_s(\lambda, Q_R)$ | Annual net revenue of the supply chain expressed in terms of $\lambda$, $Q_R$ |

This work is mainly focussed on the development of a two-echelon inventory system for the optimality of inventory decisions and shipment policies for a non-coordinated supply chain and a coordinated supply chain under quadratic price dependent demand. Numerical illustration is carried out to compare the coordinated and non-coordinated supply chain system for the optimality of decision parameters and objective function. Further, it is analysed to know the trend of inventory decisions and shipment policies by varying the dependent parameters.

## 3. Mathematical Model Formulation

In the current section, development of a mathematical model is carried out with suitable assumptions and notation, which are summarized as follows.

### *3.1 Features and Assumptions*

Based on the following features and assumptions, the proposed mathematical model is developed.

- Demand is expressed as a quadratic function of retailer's unit selling price
- Replenishment rate is instantaneous
- Replenishment batch size at the manufacturer is an integer multiple of replenishment quantity at the retailer
- Shipment quantity in each shipment from the manufacturer to retailer is equal
- No shortages are allowed

### *3.2 Model Formulation*

Two-echelon inventory system with a single-manufacturer supplying a single kind of a product to a single-retailer is considered. Demand is expressed as Quadratic function of retailer's unit selling price. Under this phenomenon, the following cost factors are considered at each echelon of the inventory situation.

#### *3.2.1 Non-Coordinated Supply Chain*

For non-coordinated supply chain, the retailer chooses his own optimal ordering quantity $Q_R$ . Next, the manufacturer chooses his own optimal number of shipments, $\lambda$ with respect to the retailer's optimal ordering quantity.

**Retailer Optimal Policy:**

Annual ordering cost of the retailer is expressed as $\frac{\left(\alpha-\beta P_R-\gamma P_R^2\right)}{Q_R} S_R$

Annual transportation cost of the retailer is expressed as $\frac{\left(\alpha-\beta P_R-\gamma P_R^2\right)}{Q_R}\tau_R$ and carrying cost is $\frac{Q_R}{2}C_R k$

Annual net revenue of the retailer is obtained by subtracting the annual ordering cost, transportation cost and carrying cost from the gross revenue.

$$\psi_R\left(Q_R\right)=\left(P_R-C_R\right)\left(\alpha-\beta P_R-\gamma P_R^2\right)-\frac{\left(\alpha-\beta P_R-\gamma P_R^2\right)}{Q_R}\left(S_R+\tau_R\right)-\frac{Q_R}{2}C_R k \tag{1}$$

**Proposition 1:** The Annual net revenue of the retailer is concave in terms of $Q_R$. The optimal replenishment quantity $Q_R^*$ is obtained by taking the first order and second order partial derivative of the annual net revenue function, as given by Eq. (2)

$$Q_R=\sqrt{\frac{2\left(\alpha-\beta P_R-\gamma P_R^2\right)\left(S_R+\tau_R\right)}{C_R k}} \tag{2}$$

**Proof:** Taking the first order and second order partial derivatives of equation (1) with respect to $Q_R$, we have $\frac{\partial}{\partial Q_R}\left(\psi_R\left(Q_R\right)\right)=0$ . With further simplification and rearranging the terms,

$$\frac{\left(\alpha-\beta P_R-\gamma P_R^2\right)}{Q_R^2}\left(S_R+\tau_R\right)=\frac{C_R k}{2} \text{ and } Q_R=\sqrt{\frac{2\left(\alpha-\beta P_R-\gamma P_R^2\right)\left(S_R+\tau_R\right)}{C_R k}}$$

$$\frac{\partial^2}{\partial Q_R^2}\left(\psi_R\left(Q_R\right)\right)=-\frac{2\left(\alpha-\beta P_R-\gamma P_R^2\right)}{Q_R^3}\left(S_R+\tau_R\right) \tag{3}$$

**Note**:From equation (3), the principal minor of Hessian matrix $H\left(Q_R\right)=\frac{\partial^2}{\partial Q_R^2}\left(\psi_R\left(Q_R\right)\right)<0$ for all values of $Q_R$.

Hence, $Q_R$ becomes optimum and $\Psi_R(Q_R)$ is strictly said to be concave.

**Manufacturer Optimal Policy:**

Annual setup cost of the manufacturer is $\frac{\left(\alpha-\beta P_R-\gamma P_R^2\right)}{\lambda Q_R}S_m$

Annual transportation cost of the manufacturer is $\frac{\left(\alpha-\beta P_R-\gamma P_R^2\right)}{Q_R}\tau_m$ and carrying cost is $\frac{(\lambda-1)Q_R}{2}C_m k$

Annual total net revenue of the manufacturer is obtained by subtracting the annual setup cost, transportation cost and carrying cost from gross revenue.

$$\psi_m\left(\lambda,Q_R\right)=\left(C_R-C_m\right)\left(\alpha-\beta P_R-\gamma P_R^2\right)-\frac{\left(\alpha-\beta P_R-\gamma P_R^2\right)}{Q_R}\left(\frac{S_m}{\lambda}+\tau_m\right)-\frac{(\lambda-1)Q_R}{2}C_m k \tag{4}$$

**Proposition 2:** For given value of $Q_R$, the optimal value of $\lambda$, $\lambda^*$always satisfies the following condition:

$$\lambda^*\left(\lambda^*-1\right)\le\frac{2\left(\alpha-\beta P_R-\gamma P_R^2\right)\left(S_m\right)}{Q_R^2 C_m k}\le\lambda^*\left(\lambda^*+1\right) \tag{5}$$

**Proof:** For given value of $Q_R$, the optimal value of $\lambda$, $\lambda^*$always satisfies the following expressions given below.

$\psi_m\left(\lambda^*\right)\ge\psi_m\left(\lambda^*-1\right)$ and $\psi_m\left(\lambda^*\right)\ge\psi_m\left(\lambda^*+1\right)$

Substituting the relevant values in eq. (4) for the condition $\Psi_m(\lambda^*)\ge\Psi_m(\lambda^*-1)$, and with further simplification and rearranging the terms, the following inequality is obtained as

$$\frac{2\left(\alpha-\beta P_R-\gamma P_R^2\right)\left(S_m\right)}{Q_R^2 C_m k}\ge\lambda^*\left(\lambda^*-1\right) \tag{6}$$

Similarly, substituting the relevant values in eq. (4) for the condition $\Psi_m(\lambda^*)\ge\Psi_m(\lambda^*+1)$,and after simplifying and rearranging the terms, the following inequality is obtained as

$$\frac{2\left(\alpha-\beta P_R-\gamma P_R^2\right)\left(A_m\right)}{Q_R^2 C_m k}\le\lambda^*\left(\lambda^*+1\right) \tag{7}$$

Combining the equations (6) and (7), the following expression is obtained as

$$\lambda^*\left(\lambda^*-1\right)\le\frac{2\left(\alpha-\beta P_R-\gamma P_R^2\right)\left(S_m\right)}{Q_R^2 C_m k}\le\lambda^*\left(\lambda^*+1\right)$$

Then, it is straightforward that the individually derived annual net revenue of the supply chain is equal to the sum of retailer's, and manufacturer's annual net revenues, i.e., $\psi_S\left(\lambda,Q_R\right)=\psi_R\left(Q_R\right)+\psi_m\left(\lambda,Q_R\right)$

### *3.2.2 Coordinated Supply Chain:*

For coordinated supply chain, when all the three parties decide to cooperate and agree to follow the optimal integrated policy, the joint annual net revenue of the retailer and manufacturer $\Psi_S(\lambda, Q_R)$ with quadratic price dependent demand is expressed as

$$\psi_S\left(\lambda,Q_R\right)=\left(P_R-C_m\right)\left(\alpha-\beta P_R-\gamma P_R^2\right)-\frac{\left(\alpha-\beta P_R-\gamma P_R^2\right)}{Q_R}\left(S_R+\frac{S_m}{\lambda}+\tau_R+\tau_m\right)-\frac{Q_R k}{2}\left(C_R+(\lambda-1)C_m\right) \tag{8}$$

**Proposition 3:** For given value of $\lambda$, the expression representing the annual net revenue of the supply chain is concave in terms of $Q_R$. The optimal ordering quantity $Q_R$ is obtained by taking the first order and second order partial derivative of the annual net revenue function, as given by equation (9).

$$Q_R = \sqrt{\frac{2\left(\alpha - \beta P_R - \gamma P_R^2\right)\left(S_R + S_m/\lambda + \tau_R + \tau_m\right)}{k\left(C_R + (\lambda - 1)C_m\right)}} \tag{9}$$

**Proof:** Taking the first order and second order partial derivatives of eq. (8) with respect to $Q_R$, and equating the first order derivative to zero, we have $\frac{\partial}{\partial Q_R}\left(\psi_S(\lambda, Q_R)\right) = 0$. With further rearranging and simplifying the terms,

$$\frac{\left(\alpha - \beta P_R - \gamma P_R^2\right)}{Q_R^2}\left(A_R + \frac{A_m}{\lambda} + \tau_R + \tau_m\right) = \frac{k}{2}\left(C_R + (\lambda - 1)C_m\right) \text{ and } Q_R = \sqrt{\frac{2\left(\alpha - \beta P_R - \gamma P_R^2\right)\left(S_R + S_m/\lambda + \tau_R + \tau_m\right)}{k\left(C_R + (\lambda - 1)C_m\right)}}$$

$$\frac{\partial^2}{\partial Q_R^2}\left(\psi_S(\lambda, Q_R)\right) = -\frac{2\left(\alpha - \beta P_R - \gamma P_R^2\right)}{Q_R^3}\left(S_R + \frac{S_m}{\lambda} + \tau_R + \tau_m\right) \tag{10}$$

**Note**:From equation (10), the principal minor of the Hessian matrix $H(\lambda, Q_R) = \frac{\partial^2}{\partial Q_R^2}\left(\psi_S(\lambda, Q_R)\right) < 0$ for all values of $\lambda$, $Q_R$ . Hence $\lambda$ and $Q_R$ become optimum. Then, $\Psi_S(\lambda, Q_R)$ is strictly said to be concave.

**Proposition 4:** For given value of $Q_R$, the optimal value of $\lambda$, $\lambda^*$ always satisfies the following condition:

$$\lambda^*\left(\lambda^* - 1\right) \le \frac{2\left(\alpha - \beta P_R - \gamma P_R^2\right)S_m}{Q_R^2 C_m k} \le \lambda^*\left(\lambda^* + 1\right) \tag{11}$$

**Proof:** For given values of $Q_R$, the optimal value of $\lambda$, $\lambda^*$ always satisfies the following expressions given below.

$\psi_S\left(\lambda^*\right) \ge \psi_S\left(\lambda^* - 1\right)$ and $\psi_S\left(\lambda^*\right) \ge \psi_S\left(\lambda^* + 1\right)$

Substituting the relevant values in equation (8) for the condition $\Psi s(\lambda^*) \ge \Psi s(\lambda^*-1)$, and after simplifying and rearranging the terms, the following inequality is obtained as

$$\frac{2\left(\alpha - \beta P_R - \gamma P_R^2\right)S_m}{Q_R^2 C_m k} \ge \lambda^*\left(\lambda^* - 1\right) \tag{12}$$

Similarly, substituting the relevant values in equation (8) for the condition $\Psi s(\lambda^*) \ge \Psi s(\lambda^*+1)$,, and after, simplifying and rearranging the terms, the following inequality is obtained as

$$\frac{2\left(\alpha - \beta P_R - \gamma P_R^2\right)S_m}{Q_R^2 C_m k} \le \lambda^*\left(\lambda^* + 1\right) \tag{13}$$

Combining equations (12) and (13), the following expression is obtained as

$$\lambda^*\left(\lambda^* - 1\right) \le \frac{2\left(\alpha - \beta P_R - \gamma P_R^2\right)S_m}{Q_R^2 C_m k} \le \lambda^*\left(\lambda^* + 1\right)$$

## 4. Numerical Illustration

In the current section, the optimality of inventory decision policies and shipment frequencies have been tested for coordinated and non-coordinated supply chain with the help of numerical data. A numerical instance is discussed here to demonstrate the proposed model.

The inventory parametric values: $S_R$ = INR 100 per order, $S_m$ = INR 300 per setup, $P_R$ = INR 160 per unit, $C_R$ = INR 140 per unit, $C_m$ = INR 100 per unit, $\tau_m$= INR 400 per shipment, $\tau_R$= INR 100 per shipment, k = 18% per year, $\alpha$ =10000, $\beta$ = 5, $\gamma$ = 0.2. Based on the program written in MATLAB as per the optimality criterion derived, the most advantageous values of resulted variables and objective function are computed for coordinated and non-coordinated supply-chain and the results are tabulated in Table 1.

From Table 1, it is observed that the number of shipments from the manufacturer to retailer decreases with supply chain coordination irrespective of the variation in demand. The annual net revenue of the supply chain will increase with coordination of the supply chain when compared with non-coordination, in all the cases of demand.

Replenishment quantity at the retailer becomes more for coordinated supply chain irrespective of the variation in demand. Further, it is identified that the yearly net income of the supply chain is less for quadratic price dependent demand, irrespective of the supply chain coordination. It is attributable to the annual sales revenue that is less for quadratic price driven demand compared to the cases of linear price dependent demand and constant demand.

**Table 1**. Optimal Values of Decision Variables and Objective Function.

| Description | Quadratic Price Dependent Demand $a = 10000, b = 5, c = 0.2, P_R = 160$ | | Linear Price Dependent Demand $a = 10000, b = 5, c = 0.0, P_R = 160$ | | Constant Demand $a = 10000, b = 0.0, c = 0.0, P_R = 160$ | |
|---|---|---|---|---|---|---|
| | Without Coordi -nation | With Coordi -nation | Without Coordi -nation | With Coordi -nation | Without Coordi -nation | With Coordi -nation |
| $D$ (in Units) | 4080.00 | 4080.00 | 9200.00 | 9200.00 | 10000.00 | 10000.00 |
| $Q_R^*$ (in units) | 254.48 | 539.84 | 382.14 | 810.64 | 398.41 | 845.15 |
| $\lambda^*$ (an integer) | 2.00 | 1.00 | 2.00 | 1.00 | 2.00 | 1.00 |
| $Q_m^*$ (in units) | 508.97 | 539.84 | 764.28 | 810.64 | 796.82 | 845.15 |
| $\psi_R^*$ (in INR) | 75187.01 | 73286.44 | 174370.05 | 171516.09 | 189960.08 | 186984.62 |
| $\psi_m^*$ (in INR) | 152091.79 | 157909.56 | 351319.55 | 360055.69 | 382609.42 | 391717.49 |
| $\psi_S^*$ (in INR) | 227278.80 | 231196.00 | 525689.59 | 531571.78 | 572569.50 | 578702.11 |

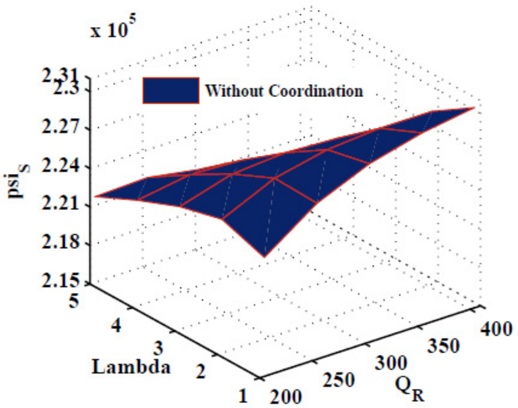


**Fig. 1** Annual Net Revenue of the Supply Chain w.r.t Replenishment Quantity and Shipment Frequency

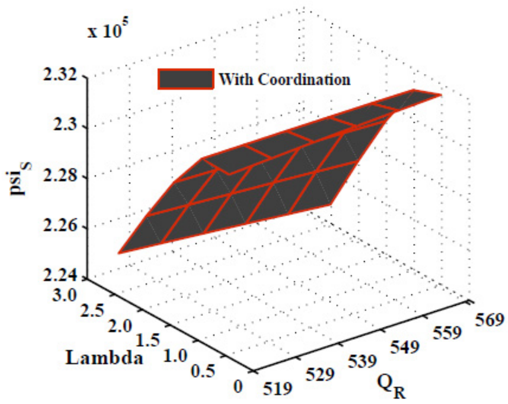


**Fig. 2** Annual Net Revenue of the Supply Chain w.r.t Replenishment Quantity and Shipment Frequency

Figure 1 and 2 show the analysis of variation of annual net revenue of the supply chain with respect to retailer's replenishment quantity and shipment frequency for non-coordinated and coordinated supply chain. From these figures, it is evident that the annual net revenue of the supply chain assumes concavity in its shape with respect to simultaneous variation in retailer's replenishment quantity and the number of shipments from the manufacturer to retailer. Further, the sensitivity analysis is carried out to analyze the influence of model parameters over the optimality of decision variables and objective function. Table 2 and 3 show the analysis of variation of replenishment quantity, shipment frequency and annual revenue of the respective entities and the supply chain for coordinated and non-coordinated chain.

**Table 2: SENSITIVITY ANALYSIS**

| Para -meter | | Without Coordination | | | | | | | With Coordination | | | | | | % Increase |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | D | $Q_R^*$ | $\lambda$ | $Q_m^*$ | $\psi_R^*$ | $\psi_m^*$ | $\psi_S^*$ | $Q_R^*$ | $\lambda$ | $Q_m^*$ | $\psi_R^*$ | $\psi_m^*$ | $\psi_S^*$ | in $\psi_S^*$ |
| $S_m$ | - 40% | 4080 | 254.5 | 1 | 254.5 | 75187.0 | 153901.2 | 229088.2 | 502.6 | 1 | 502.6 | 73644.0 | 158491.4 | 232135.4 | 1.33 |
| | - 20% | 4080 | 254.5 | 1 | 254.5 | 75187.0 | 152939.2 | 228126.2 | 521.5 | 1 | 521.5 | 73464.0 | 158193.3 | 231657.3 | 1.55 |
| | +20% | 4080 | 254.5 | 2 | 509.0 | 75187.0 | 151610.8 | 226797.8 | 557.5 | 1 | 557.5 | 73111.4 | 157638.5 | 230749.8 | 1.74 |
| | +40% | 4080 | 254.5 | 2 | 509.0 | 75187.0 | 151129.8 | 226316.9 | 574.7 | 1 | 574.7 | 72938.9 | 157378.6 | 230317.4 | 1.77 |
| $S_R$ | - 40% | 4080 | 227.6 | 2 | 455.2 | 75864.1 | 151292.8 | 227156.8 | 527.7 | 1 | 527.7 | 73713.8 | 157787.9 | 231501.7 | 1.91 |
| | - 20% | 4080 | 241.4 | 2 | 482.8 | 75516.1 | 151732.3 | 227248.5 | 533.8 | 1 | 533.8 | 73498.2 | 157849.8 | 231348.0 | 1.80 |
| | +20% | 4080 | 266.9 | 1 | 266.9 | 74874.0 | 152499.5 | 227373.6 | 545.8 | 1 | 545.8 | 73078.3 | 157967.4 | 231045.7 | 1.62 |
| | +40% | 4080 | 278.8 | 1 | 278.8 | 74574.9 | 152955.1 | 227530.0 | 551.7 | 1 | 551.7 | 72873.6 | 158023.3 | 230897.0 | 1.48 |
| $C_m$ | - 40% | 4080 | 254.5 | 2 | 509.0 | 75187.0 | 316207.9 | 391394.9 | 539.8 | 1 | 539.8 | 73286.4 | 321109.6 | 394396.0 | 0.77 |
| | - 20% | 4080 | 254.5 | 2 | 509.0 | 75187.0 | 234149.9 | 309336.9 | 539.8 | 1 | 539.8 | 73286.4 | 239509.6 | 312796.0 | 1.12 |
| | +20% | 4080 | 254.5 | 1 | 254.5 | 75187.0 | 70377.3 | 145564.3 | 539.8 | 1 | 539.8 | 73286.4 | 76309.6 | 149596.0 | 2.77 |
| | +40% | 4080 | 254.5 | - | - | 75187.0 | - | - | 539.8 | 1 | 539.8 | 73286.4 | -5290.4 | 67996.0 | - |
| $C_R$ | - 40% | 4080 | 328.5 | - | - | 305112.5 | - | - | 696.9 | 1 | 696.9 | 303640.3 | -69378.0 | 234262.4 | - |
| | - 20% | 4080 | 284.5 | 1 | 284.5 | 190104.1 | 38922.1 | 229026.1 | 603.6 | 1 | 603.6 | 188404.1 | 44228.1 | 232632.2 | 1.57 |
| | +20% | 4080 | - | - | - | - | - | - | 492.8 | 1 | 492.8 | -41747.0 | 271644.6 | 229897.6 | - |
| | +40% | 4080 | - | - | - | - | - | - | 456.2 | 1 | 456.2 | -156716.7 | 385420.3 | 228703.5 | - |
| $\tau_m$ | - 40% | 4080 | 254.5 | 2 | 509.0 | 75187.0 | 154657.0 | 229844.0 | 489.5 | 1 | 489.5 | 73765.2 | 158699.2 | 232464.4 | 1.14 |
| | - 20% | 4080 | 254.5 | 2 | 509.0 | 75187.0 | 153374.4 | 228561.4 | 515.3 | 1 | 515.3 | 73523.8 | 158290.9 | 231814.7 | 1.42 |
| | +20% | 4080 | 254.5 | 2 | 509.0 | 75187.0 | 150809.2 | 225996.2 | 563.3 | 1 | 563.3 | 73053.6 | 157550.7 | 230604.2 | 2.04 |
| | +40% | 4080 | 254.5 | 2 | 509.0 | 75187.0 | 149526.6 | 224713.6 | 585.9 | 1 | 585.9 | 72825.3 | 157210.9 | 230036.2 | 2.37 |
| $\tau_R$ | - 40% | 4080 | 227.6 | 2 | 455.2 | 75864.1 | 151292.8 | 227156.8 | 527.7 | 1 | 527.7 | 73714.0 | 157787.9 | 231501.8 | 1.91 |
| | - 20% | 4080 | 241.4 | 2 | 482.8 | 75516.1 | 151732.3 | 227248.5 | 533.8 | 1 | 533.8 | 73498.2 | 157849.8 | 231348.0 | 1.80 |
| | +20% | 4080 | 266.9 | 1 | 266.9 | 74874.0 | 152499.5 | 227373.6 | 545.8 | 1 | 545.8 | 73078.3 | 157967.4 | 231045.7 | 1.62 |
| | +40% | 4080 | 278.8 | 1 | 278.8 | 74574.9 | 152955.1 | 227530.0 | 551.7 | 1 | 551.7 | 72873.6 | 158023.3 | 230897.0 | 1.48 |

**Table 3: SENSITIVITY ANALYSIS**

| Para -meter | | Without Coordination | | | | | | | With Coordination | | | | | | % Increase |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | D | $Q_R^*$ | $\lambda$ | $Q_m^*$ | $\psi_R^*$ | $\psi_m^*$ | $\psi_S^*$ | $Q_R^*$ | $\lambda$ | $Q_m^*$ | $\psi_R^*$ | $\psi_m^*$ | $\psi_S^*$ | in $\psi_S^*$ |
| $P_R$ | - 40% | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| | - 20% | - | - | - | - | - | - | - | 659.2 | 1 | 659.2 | -83149.7 | 236868.1 | 153718.3 | - |
| | +20% | 1667 | 162.7 | 2 | 325.4 | 82595.0 | 59587.2 | 142182.2 | 345.1 | 1 | 345.1 | 81380.1 | 63306.1 | 144686.2 | 1.76 |
| | +40% | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $\alpha$ | - 40% | 80 | 35.6 | 2 | 71.3 | 702.0 | 1644.5 | 2346.5 | 75.6 | 1 | 75.6 | 436.0 | 2459.1 | 2895.1 | 23.38 |
| | - 20% | 2080 | 181.7 | 2 | 363.4 | 37021.1 | 75268.7 | 112289.8 | 385.4 | 1 | 385.4 | 35664.1 | 79422.6 | 115086.7 | 2.49 |
| | +20% | 6080 | 310.7 | 2 | 621.3 | 113771.4 | 229639.8 | 343411.2 | 659.0 | 1 | 659.0 | 111451.3 | 236741.8 | 348193.1 | 1.39 |
| | +40% | 8080 | 358.1 | 2 | 716.3 | 152575.2 | 307567.8 | 460143.1 | 759.7 | 1 | 759.7 | 149900.6 | 315754.9 | 465655.6 | 1.20 |
| $\beta$ | - 40% | 4400 | 264.3 | 2 | 528.5 | 81340.3 | 164464.4 | 245804.7 | 560.6 | 1 | 560.6 | 79366.6 | 170506.0 | 249872.6 | 1.65 |
| | - 20% | 4240 | 259.4 | 2 | 518.9 | 78262.5 | 158276.1 | 236538.6 | 550.3 | 1 | 550.3 | 76325.0 | 164206.8 | 240531.8 | 1.69 |
| | +20% | 3920 | 249.4 | 2 | 498.9 | 72114.0 | 145911.8 | 218025.8 | 529.2 | 1 | 529.2 | 70251.1 | 151614.3 | 221865.4 | 1.76 |
| | +40% | 3760 | 244.3 | 2 | 488.6 | 69043.6 | 139736.3 | 208779.9 | 518.2 | 1 | 518.2 | 67219.1 | 145321.3 | 212540.4 | 1.80 |
| $\gamma$ | - 40% | 6128 | 311.9 | 2 | 623.8 | 114700.6 | 231506.4 | 346207.0 | 661.6 | 1 | 661.6 | 112371.4 | 238636.3 | 351007.7 | 1.39 |
| | - 20% | 5104 | 284.6 | 2 | 569.3 | 94907.3 | 191735.8 | 286643.0 | 603.8 | 1 | 603.8 | 92781.5 | 198242.8 | 291024.3 | 1.53 |
| | +20% | 3056 | 220.2 | 2 | 440.5 | 55569.8 | 112626.3 | 168196.1 | 467.2 | 1 | 467.2 | 53925.0 | 117661.3 | 171586.3 | 2.02 |
| | +40% | 2032 | 179.6 | 2 | 359.2 | 36114.2 | 73440.7 | 109555.0 | 381.0 | 1 | 381.0 | 34773.0 | 77546.4 | 112319.4 | 2.52 |
| k | - 40% | 4080 | 328.5 | 2 | 657.1 | 76632.5 | 154595.6 | 231228.1 | 696.9 | 1 | 696.9 | 75160.4 | 159102.0 | 234262.4 | 1.31 |
| | - 20% | 4080 | 284.5 | 2 | 569.0 | 75864.1 | 153264.5 | 229128.6 | 603.6 | 1 | 603.6 | 74164.1 | 158468.1 | 232632.2 | 1.53 |
| | +20% | 4080 | 232.3 | 2 | 464.6 | 74574.9 | 151031.6 | 225606.5 | 492.8 | 1 | 492.8 | 72493.0 | 157404.6 | 229897.6 | 1.90 |
| | +40% | 4080 | 215.1 | 2 | 430.2 | 74012.1 | 150056.6 | 224068.6 | 456.2 | 1 | 456.2 | 71763.3 | 156940.3 | 228703.5 | 2.07 |

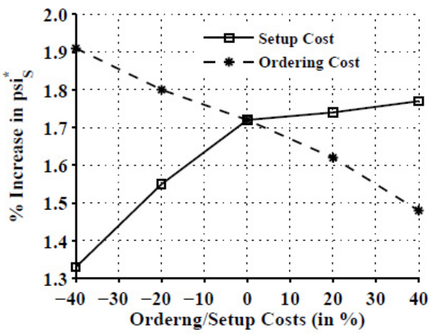


**Fig. 3** % Increase in $\psi_s^*$ due to Coordination w.r.t. Ordering/Setup Costs

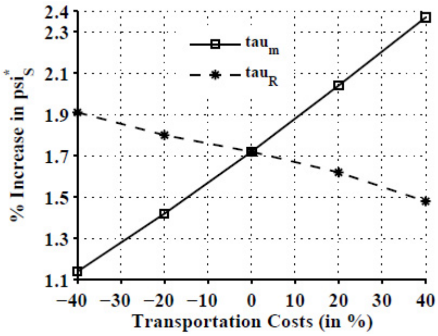


**Fig. 4** % Increase in $\psi_s^*$ due to Coordination w.r.t. Transportation Costs

Figure 3 and Table 2 are showing the analysis of change of percentage increase in yearly net revenue of the supply chain because of coordination with respect to retailer's ordering cost and setup cost of the manufacturer. It is observed that as the setup cost increases, the percentage increase in annual net revenue of the supply chain will go high. It is because the rate of decrease in annual net revenue at the manufacturer is less for coordinated chain rather than non-coordinated chain. Whereas, the percentage increase in annual net revenue of the supply chain due to coordination decreases with respect to change in cost of ordering. It is because the rate of decrease in net revenue of the coordinated chain is more than the rate of increase in net revenue of the non-coordinated chain.

Figure 4 and Table 2 are showing the study of change of percentage increase in yearly net income of the supply chain relating to transportation cost of manufacturer and retailer. It is noted that as the transportation cost of the manufacturer increases, the percentage increase in annual net revenue of the supply chain increases. It is due to the fact that the rate of increase in total relevant cost at the manufacturer is less for coordinated chain rather than non-coordinated chain. It is also found that as the transportation cost of the retailer increases, the percentage increase in annual net revenue of the chain decreases. It is due to the fact that the rate of increase in total relevant cost at the retailer is more for coordinated chain rather than non-coordinated chain. Also, from table 3, it is evident that the percentage increase in annual net revenue of the supply chain due to coordination decreases with respect to the constant 'alpha', whereas increases with respect to constants 'beta' and 'gamma'.

## 5. Conclusions

In this work, a two-echelon supply chain consisting of a single manufacturer supplying a single kind of product to a single retailer is considered. A mathematical model for a two-echelon inventory system is developed, under quadratic price dependent demand for a coordinated and non-coordinated supply chain. Computer program is written in MATLAB as per the optimality criteria derived and the model is solved with the help of numerical data. The optimality of inventory decisions, shipment policies and the annual net revenue of the supply chain are demonstrated for coordinated and non-coordinated chain. Also, the sensitivity analysis is carried out. From the research findings of this work, it is concluded that the annual net revenue of the supply chain is less for quadratic price dependent demand in comparison to linear price dependent demand and constant demand. Retailer's replenishment quantity at the retailer is also less for quadratic price dependent demand. Shipment frequency remains for quadratic price dependent demand, linear price dependent demand and constant demand.

From the sensitivity analysis of the model, it is also found that the variation in model parameters has significant influence over the optimality of replenishment decisions, shipment frequencies and the annual net revenue of the retailer, manufacturer and supply chain, for both the cases of coordination. Due to coordination, with respect to increase in manufacturer's setup cost, unit cost, transportation cost, and constants beta, gamma and the interest rate, the percentage increase in annual net revenue of the supply chain increases, Where as it decreases with ordering and transportation cost of the retailer and the constant beta.

Finally, it is concluded that the current work comply with the practical aspects of business scenario, where the end demand is price sensitive. Especially, for consumer goods, durable goods etc., in order to study the optimality of inventory and shipment policies, the current model can be used. Although, this work endeavors to address some managerial inferences, still it is a well-known fact that the scope of the multi-echelon inventory supply chain models is unlimited. The present work may be extended by considering multi-channel multi-echelon supply chain.

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