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# Optimization of Pulse Sequences in MRI Scheme 

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#### Abstract

Magnetic resonance imaging (MRI) has a wide range of applications towards imaging the human body. In this work we have solved the Bloch equations for different magnetic field gradients along the transverse direction. We have modified the magnetic field components based on the relaxation terms and solved the field gradient as well as the field components for both off -pulse and on -pulse configurations. In particular we focus on different pulse sequences and optimize them to realize the best possible output. We have analyzed the field components along transverse direction because the rotation of the object to form the image by emitting signal is along the xy plane.


## 1. Introduction

The optimization of pulses during the scanning process in magnetic resonance imaging is very essential for getting an image from the object or human body. This has versatile applications including imaging technique in medical research field. In order to optimize the output we can vary the pulse duration and the amplitude with specific conditions [1]. There are several processes to change the input pulse which affects the pattern of output image. If we set the repetition time longer, then the image quality will be brightened. We can change brightness of the output signal by increasing or decreasing the value of repetition time. The alternative way to change the pulse sequence is to vary the amplitude of echo time as well as band width of the pulses. In the applied pulse sequences, if we increase the field strength up to 3 Tesla then it can be used in the angiography analysis. In magnetic resonance measurements the signal to noise ratio will increase with the increase in magnetic field [2]. The appropriate pulse sequences helps to find the genetic nature in any living object. Another application which has a strong dependence on this optimization of pulse sequence is chemical exchange property. The saturation transfer of this exchange is analyzed by the detailed optimization of pulses. In the measurement of cardiac imaging, the gradient pulse [1-3] wave helps to analyze the output signal. It has a great importance in the imaging process of brain which is the part of functional magnetic resonance imaging. Here the response of rate of flow of blood in the human body is studied by this technique. It also helps in the small tip excitation study [4]. One of the main advantages of this work is we can get the output image with various brightness/contrast and this can be manipulated by varying different parameters in a given pulse sequence.
2. Theoretical Formalism

### 2.1. Bloch Equations

In this section we explain the Bloch equations explicitly. These nuclear components act as a time dependent function in the presence of various relaxation times [5-6]. To observe the MRI we need to understand the effect of magnetic field interactions when we are considering a weak magnetic field $M(t)$ in the presence of a large external magnetic field $B(t)$. The angle between two different magnetic fields is $\alpha$. The large external field is applied along the $z$ direction. The torque can be expressed as,

$$
\begin{equation*}
\frac{d\{\vec{J}(t)\}}{d t}=\vec{M}(t) \times \vec{B}(t) \tag{1}
\end{equation*}
$$

Now we have solved the above Bloch equation based on the magnetic components along three axes. The vertical component will be,

$$
\begin{equation*}
M_{z}(t)=M_{0} \cos \alpha \tag{2}
\end{equation*}
$$

The transverse components are,

$$
\begin{equation*}
M_{x y}(t)=M_{0} \sin \alpha e^{j \phi} e^{-j \gamma B 0 t} \tag{3}
\end{equation*}
$$

Where $\phi$ is the angle between the projection of weak moment in the xy plane and $x$-axis, where the direction must be towards the $y$ axis. Thus we can see that the component along vertical direction of the weak field will not change but the other two components of this moment will rotate with a certain frequency. The value of this frequency is $\gamma B_{0}$ radians and it can goes upto $M_{0} \sin \alpha$ by amplitude.


Figure 1. Coordinate system illustrating Bloch equation
Now we try to produce the disturbance so that we can get the non- zero value of $\alpha$ for emitting the resonant signal. Hence we are giving an external magnetic field $E(t)$ which is greater than $B(t)$. We have to apply this field for a short time and the rotation of the field should be matched with Larmor frequency. In principle when $t=0$, there will be no field components along transverse plane. But in reality, this picture will change due to the disturbance of net magnetic moment at an extent. So small amount of xy components should be considered with small frequency.

$$
\begin{equation*}
\frac{d \vec{M}(t)}{d t}=\gamma \vec{M}(t) \times[\vec{E}(t) \times \vec{B}(t)] \tag{4}
\end{equation*}
$$

Finally we have solved this expression and determined the transverse and vertical components.

$$
\begin{align*}
& \qquad M_{x y}(t)=2 \frac{-E_{0} M_{0} \cos \alpha}{B_{0}} e^{-j\left(-\gamma B_{0} t+\theta\right)} \frac{-B_{0}}{E_{0}} \tan \alpha e^{j(\varphi-\theta)}  \tag{5}\\
& \text { or, } \quad M_{x y}(t)=2 M_{0} \sin \alpha\left(e^{j\left(-\gamma B_{0} t+\phi\right)}\right)  \tag{6}\\
& \text { and } \vec{M}_{z}(t)=\gamma\left\{M_{0} \sin \alpha E_{0} \sin (\theta-\phi) t\right\}+2 M_{0} \cos \alpha \tag{7}
\end{align*}
$$

Here we can see that the amplitude of the magnetic moment is exactly same as the amplitude when there is no additional radio frequency magnetic field. But in this case, we are getting the image of the part of inserted object due to resonance.

### 2.2. Inclusion of relaxation terms

The internal weak magnetic spin of the object what we entered in the MRI will evolve over time and finally relaxes, during that relaxation time they will emit some energy in terms of varying radio frequency signal. This can be detected by the receiver of the MRI. Then this signal can be used to construct a image. The rate of decay along vertical component will be (1/T1). This means after the time constant T 1 spin will recover $63 \%$ of its growth of magnetization vector of its initial value. When this signal reduces by an amount of $37 \%$ of its original value, then it is characterized by the parameter T2. It comes from the transverse plane. We can write the modified equation below,

$$
\begin{equation*}
\frac{d M}{d t}=\vec{M}(t) \times \gamma \vec{B}(t)-R \tag{8}
\end{equation*}
$$

Here R is the relaxation matrix which is used to transform the spin direction. In other words we can say it is the transformation matrix. We have already determined the general expression of Bloch equation for both the relaxation mechanisms. Then we have taken only the components along $x$ and $y$ directions.

$$
\begin{align*}
\frac{d M_{x}}{d t} & =\left[-\omega M_{y}-\gamma B_{y}^{r} M_{z}\right]-\frac{M_{x}(t)}{T 2}  \tag{9}\\
\frac{d M_{y}}{d t} & =\left[\gamma M_{z} B_{x}^{r}+\omega M_{x}\right]-\frac{M_{y}(t)}{T 2} \tag{10}
\end{align*}
$$

And the z term is from the above expression is,

$$
\begin{equation*}
\frac{d M_{z}}{d t}=\gamma\left[M_{x} B_{y}^{r}-M_{y} B_{x}^{r}\right]-\frac{M_{z}(t)-M_{0}}{T 1} \tag{11}
\end{equation*}
$$

### 2.3. Mathematical formulations

Now we restrict the above equations at the time of applying the pulse. There we dealt with two cases. In first case, we are discussing with single pulse. When we give the pulse along any one of the axes, then it is known as single pulse [7]. Here we have given along x direction only. One condition should be satisfied that the pulse width should be much smaller than the time constant T1 and T2. Now as we apply the pulse along x axis, the magnetic field along y direction $\left(\mathrm{B}_{\mathrm{y}}\right)$ and the oscillation frequency with respect to z axis can be neglected. The other thing is time constant consideration. As the pulse is very small, so we can neglect the relaxation term [8]. After putting these conditions into above expressions, we can get the following equations.

$$
\begin{align*}
\frac{d M_{z}}{d t} & =-\gamma M_{y} B_{x}^{r}(t)  \tag{12}\\
\frac{d M_{x}}{d t} & =0  \tag{13}\\
\frac{d M_{y}}{d t} & =\gamma M_{z} B_{x}^{r}(t) \tag{14}
\end{align*}
$$

The solution of the above equations are,

$$
\begin{align*}
& M_{z}(t)=-M_{0} \cos \left(\omega_{x} t\right)  \tag{15}\\
& M_{y}(t)=-M_{0} \sin \left(\omega_{x} t\right)  \tag{16}\\
& M_{x}(t)=0 \tag{17}
\end{align*}
$$

When there will be no pulse during scanning, then it is known as free precession. In this condition, magnetic field along transverse plane will be zero. We have to neglect the value of $B_{x}$ and $B_{y}$. After putting these considerations into modified Bloch equations i.e. (9), (10) and (11), then we get the following equations.

$$
\begin{align*}
M_{z}(r, t) & =M_{0}(r)\left[1-e^{-\frac{t}{T 1}}\right]+M_{z}(r, 0) e^{-\frac{t}{T 1}}  \tag{18}\\
M_{x} & =M_{0} \cos \omega t e^{-\frac{t}{T 2}}  \tag{19}\\
M_{y} & =M_{0} \sin \omega t e^{-\frac{t}{T 2}} \tag{20}
\end{align*}
$$

## 3. Optimization of different pulse sequences

3.1. Two separate conditions for one single output pulse

In this part, we have applied the pulse in two different cases and also we have compared the output characteristic of the applied pulse. In the first case we have taken one square pulse in a way so that the time duration of the pulse is fixed when there is a field and there is a change in the evolution time. In the second case we have taken the output pulse in such a way that the time duration of the pulse varies continuously both for on and off- field condition of the pulse.

### 3.1.1. Application of pulse 1



Figure 2. Structure of pulse 1 in the input
Here one single on and off pulse the components along z direction are given.

$$
\begin{equation*}
M_{z}(t)=M_{0} \cos \left(\omega_{x} t\right)+M_{0}(r)\left[1-e^{-\frac{t}{T 1}}\right]+M_{z}(r, 0) e^{-\frac{t}{T 1}} \tag{21}
\end{equation*}
$$

The components along x and y direction for single on and off pulses are given below.

$$
\begin{align*}
& M_{x}(t)=0+M_{0} \cos \left(\omega_{x} t\right) e^{-\frac{t}{T 2}}  \tag{22}\\
& M_{y}(t)=M_{0} \sin \left(\omega_{x} t\right)+M_{0} \sin \left(\omega_{x} t\right) e^{-\frac{t}{T 2}} \tag{23}
\end{align*}
$$

### 3.1.2. Application of pulse 2



Figure 3. Structure of pulse 1 in the input

### 3.2. Consideration of feedback term for two conditions

Here we have modified the applied pulse by considering the feedback terms. Here every time the field vector takes its previous value as a feedback term when the next pulse is applied. The rotation of the entering object will be in the transverse plane. So our main focus is on the xy plane. Next we plotted the pulse duration versus magnetic vector for getting a good calibration in both the directions x and y with two different pulse conditions as mentioned above. In this case we took the magnetic components like,
$M_{x} 2=M_{x} 1+M_{x} 2 ; M_{x} 3=M_{x} 2+M_{x} 3 ; M_{x} 4=M_{x} 3+M_{x} 4 \ldots$ and so on.

## 4. Results and discussions

4.1. Output characteristics of applied pulse with feedback terms

The x and y component of magnetization vector with off pulse time (evolution time) for the above condition (pulse 1) is shown in figures 4 and 5.


Figure 4. Magnetic field component along x axis versus off pulse time.


Figure 6. Magnetic field component along x axis versus off pulse time.

Figure 5. Magnetic field component along y axis versus off pulse.


Figure 7. Magnetic field component along y axis versus off pulse time.

In Figure 4, the magnetic vector is increasing exponentially for the first few microseconds but after a certain limit it is going to be stable. In this case the signals coming from the $y$ direction of the inserted object can be analyzed easily. The output image of the object or human body can be studied only along y direction in the magnetic resonance imaging machine. In Figure 5, the image formation can be found but after a limited time it will disappear. It is not stable for long time. So the signal can be modified partially by the new condition. For pulse 2 we get the following output curves along rotating plane. From the Figures 6 and 7 we can qualitatively say that this is the improved output pulse in comparison with the previous one. Here the nature of the calibration is exponentially increasing.

## 5. Conclusions

In this work, we have worked on the optimization of pulses. Here we have developed the final pulse calibration curve from the basic quantum Bloch equations and solved the magnetic field components both in transverse and vertical plane with the function of time. We have modified the solutions of the magnetic fields with the inclusion of relaxation terms. The components for on pulse and off pulse or free precession have determined. During the application of input pulse we have given two separate pulses with different conditions. In the application of second pulse with feedback conditions we are able to get better result for the field along both the axes. We get a better stability along both directions of planes. Here for pulse 1 we get the stable calibration along $x$ direction and for pulse 2 we get the stability along both directions. Thus pulse optimization has introduced in the input as well as output for the solution of finding the better signal. This in turn helps to find the magnetic field components in the field of magnetometry applications.

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