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# Performance Analysis of Relay Assisted Multihop Coherant OFDM System over Malaga Distribution with Pointing Errors

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**Abstract:** This paper presents a multi-hop coherent orthogonal frequency division multiplexing free space optical communication system (OFDM-FSO) model which includes a relay transmission mode of decode and forward (DF) over Malaga disrubution (M) and we analyse the results by deriving mathematical expressions for average bit error rate (BER) and outage probability. And the FSO link is characterized by path loss, pointing errors and atmospheric turbulence and also weather conditions like clear air, haze, light fog have been considered in the channel model. The BER and outage probability of the proposed system are examined including the effects of pointing errors and weather conditions. From this work, we can justify that with multi-hop configuration coverage area and signal power improves which bolsters the usage of relay in FSO communication for efficient and reliable system design.

**Keywords:** Pointing error, M-distribution, orthogonal frequency division multiplexing (OFDM), multi-hop system, outage probability

## 1 Introduction

In the present world, communication system offering huge bandwidth and high data rate is the cynosure for all the researchers, FSO is a elixir for the researchers. Because of its unique characteristics like cost-effective, secure and huge bandwidth access [1–6] . But atmospheric conditions causes path loss and pointing errors which acts as a hindrance for effective FSO communication system design. Hence researchers are focussing in reducing these errors, if not FSO communication will become unreliable over a distances of 1 km or longer. P Pointing error caused by the misalignment of the transmitter and receiver, the path loss due to the absorption and scattering of various particles in the atmosphere channel, and the atmosphere turbulence effect which occurs as a result of rapid fluctuations of received signal, all have a adverse impact on the FSO link performance and should be lessened for efficient and reliable FSO system design.

Relays in FSO system improves signal power to a fair amount, lot of research has been carried in implementing either parallel (i.e. cooperative diversity) or serial relaying (i.e. multi-hop transmission) techniques. Again serial relaying is divided into amplify and forward (AF) and decode and forward (DF) transmission strategies. Multihop or serial transmission efficiently joins all the relays serially and hence increase the coverage area . In parallel relaying multi-laser transmitter apertures are focused on the relay nodes. However in [7] FSO system with both serial and parallel relaying are together implemented. They proposed that the outage performance of the multihop parallel FSO system does exceptionally well then compared to individual performance of serial or parallel relaying. But individually serial relaying outperforms that of parallel.

OFDM is enticing modulation technique for wireless communication system [8] which uses an efficient way of a transmitting data parallely rather than serially using multiple narrow band sub-carriers. The sub-carriers are orthogonal to each other, and thus, the inter symbol interference (ISI) between sub-carriers are minimum, so the signal separation at the receiver side using OFDM is effective and prone to less attenuation and minimum multi path fading.

To evaluate the performance of FSO system, an precise mathematical model to describe the optical channel characteristics with respect to the atmospheric turbulence is required. From the literature survey, we have log-normal (LN), GammaGamma (GG), K and negative exponential distributions. The LN model is restricted to weak turbulence regime. In [9], the outage probability for multihop FSO system with DF protocol has been analysed

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based on the LN distribution. Compared with LN distribution, GG distribution can be used for all the turbulence regimes. The outage probability and the average symbol error rate (ASER) of the multi-hop DF FSO links over the GG turbulence channels considering the path loss and pointing errors have been studied systematically in [10]. But LN, GG, K and negative exponential models mentioned above are special cases of a more generic model named M distribution or Malaga distribution, which is proposed in [11]. The uniqueness in M -distribution is it covers all the channel conditions from weak to strong turbulence and it is capable of characterizing most of the existing atmospheric turbulence models [12–15] shown in Table 1.

**Table 1:** List of existing distribution models for atmospheric optical communications and generation by using the proposed M distribution model [11].

| Distribution model | conditions |
|---|---|
| Gamma | $\rho = 0, \gamma = 0$ |
| Gamma-gamma | $\rho = 1, \gamma = 0, \Omega' = 1$ |
| Lognormal | $\rho = 0, \gamma = 0, var[\lvert U_L \rvert] = 0$ |
| K distribution | $\rho = 0, \Omega = 0$ or $\beta = 1$ |
| Exponential distribution | $\rho = 0, \Omega = 0$ |

Motivated by the above analysis, there are some works, in [16] amplify and forward (AF) and decode and forward (DF) relay scheme for serial communication under the strong turbulence, and optimal performance was by DF relay scheme, in [17] multi-hop coherent OFDM system is analysed including path loss and pointing errors with gamma gamma distribution channels in [18] relay aided FSO system based on binary phase shift keying (BPSK) including pointing errors, path loss and atmospheric turbulence models. In this work, we extend the outage probability and BER performance presented in [17] with M distribution including the effects of atmospheric turbulence. Work is novel with the usage of M distribution, OFDM, serial relay which are the harbinger in present wireless communication.

The remainder of the paper is arranged as follows. In Section 2, the system model with relays are discussed. In Section 3, we introduce the FSO channel model including path loss, pointing errors and atmospheric turbulence. In Section 4, we derived the expressions for average BER and outage probability of the considered system in Section 5. Section 6 describes the numerical results with graphical analysis. Ultimately conclusion in Section 7.

## 2 System model

A coherent FSO communication system with relays is illustrated in Figure 1. We consider source node (S) communicating with destination node (D) at a distance of some X kilometres via $N + 1$ serial paths ($N$). Let $L$ denotes the total number of hops in each serial path ($N$), hence there are $L - 1$ relays in each serial path ($N$). Multi-hop plays a pivotal role in accruing the coverage capability of the transmitter, and individual relays are able to monitor and transmit the signal and absolve the problem of obstructions that occur in between transmitter and receiver. Relays acts as a intermediate between transmitter and receiver or special environment and so

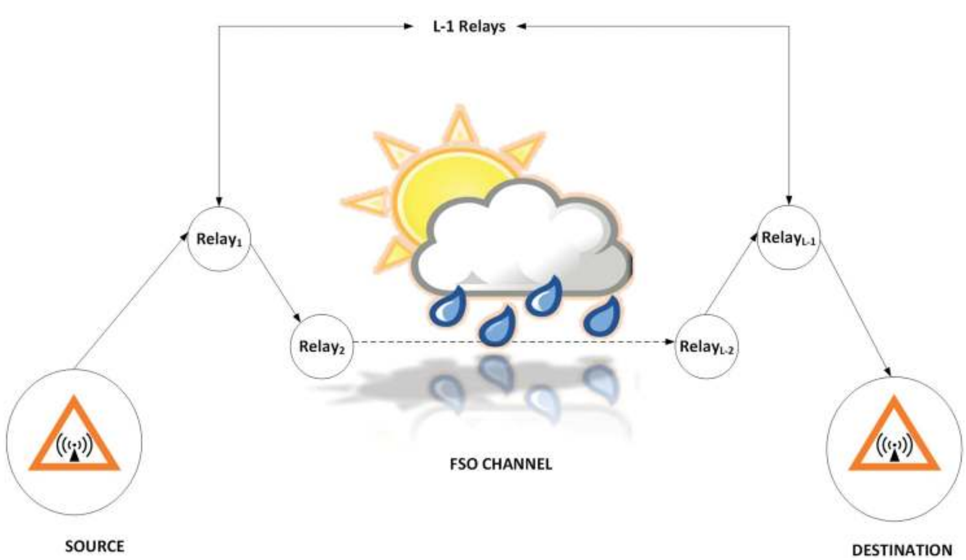


**Figure 1:** Schematic of Multi-hop DF FSO system.

on. Serial Relaying are of two types decode and forward [DF] and Amplify and forward [AF] in general. And DF relay mode is more desirable then AF because it amplifies signal as well as noise to the next relay node which is avoided in DF. As mentioned above relay nodes are composed of coherent transmitting and receiving antennas shown in Figure 1.

The system under consideration is employed with OFDM modulation, OFDM signal transmitted by the laser enters atmosphere and gets attenuated by its effects and is received at the receiver side by the photo detector (PD), as illustrated in Figure 2. OFDM is a form of multicarrier modulation technique [19, 20]. It uses multiple sub-carriers within the same single channel. It employs a large number of orthogonal sub-carriers. Each is modulated by digital modulation techniques. The message bits are first transformed from serial to parallel form and then modulated before being converted to symbols. The total data rate is same as that of conventional single sub-carrier modulation schemes having same bandwidth. Inverse Fast Fourier Transform (IFFT) is done at the transmitter and Fast Fourier Transform (FFT) at the receiver to retrieve the data. The transmitted power gets reduced as it travels through the medium, Noise $n(t)$ especially additive white Gaussian noise (AWGN) gets added while it gets transmitted through the free atmosphere. At the (PD) of the received is given by $y_i = I_i R_i x_i + n_i$, where $I_i$ is the channel model, $R_i$ indicates the responsivity of the $i$th receiver, $x_i$ is the source signal, and $n_i$ is the additive white Gaussian noise.

## 3 Channel model

In the present paper, the optical channel model $I_{ij}$ is considered as product of $I_a$, $I_p$ and $I_l$ which is given below [18]:

$$I_i = I_l I_a I_p. \tag{1}$$

Here, $I_l$ represents atmospheric loss, $I_a$ represents atmospheric turbulence, $I_p$ represents pointing errors

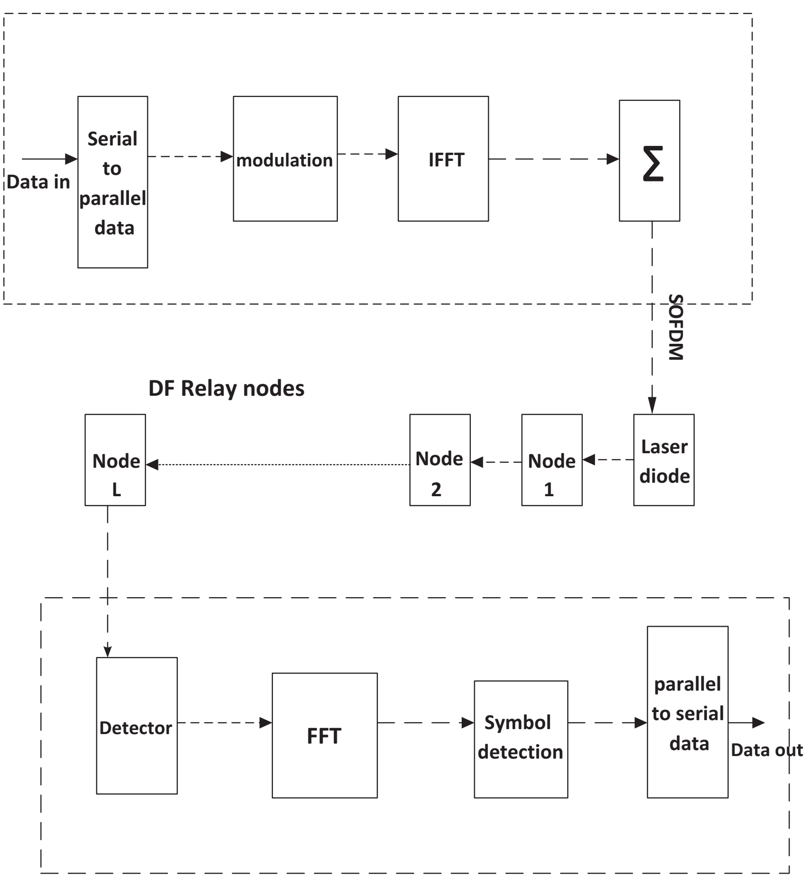


**Figure 2:** Schematic of Multi-hop DF FSO system.

## 3.1 Atmospheric loss

Where $I_l$ represents the atmospheric loss modelled by the Beer–Lamberts law as [18] and $I_P$ represents pointing errors which is discussed further

$$I_l = \exp(-\sigma L), \tag{2}$$

where $\sigma$ is the attenuation coefficient, $L$ is the link length. The $\sigma$ under different weather conditions at a wavelength of 1,530 nm is chosen from [18] and the values adopted can be found in Table 2 and link length is 10 km

**Table 2:** Attenuation coefficients for different weather conditions [18].

| Weather condition | Attenuation $\sigma$(dB/KM) |
|---|---|
| Very clear air | 0.0647 |
| Clear air/drizzle | 0.2208 |
| Haze | 0.7360 |
| Light fog | 4.2850 |

## 3.2 Atmospheric turbulence induced fading

In this paper, channel is exhibited using the summed up FSO channel known as the M-distribution channel. The uniqueness in M-channel is it covers all the channel turbulence conditions and it is capable of deriving most of the at present existing turbulence models shown in Table 2 for instance gamma, gamma, negative exponential, K dispersion, lognormal models [11, 21, 22]. And furthermore the impact of misalignment blunders amongst transmitter and the receiver due to non-line of sight pathway otherwise called pointing mistakes whose lessening is essential for proficient FSO system.

The probability density function (PDF) of the Malaga-distribution turbulence is given by [23]

$$f_{I_i}(I_i) = A\sum_{k=1}^{b} a_k I_i^{\frac{\alpha+k}{2}-1} K_{\alpha-k}\left(2\sqrt{\frac{\alpha\beta I_i}{\gamma\beta+\Omega}}\right) \tag{3}$$

where

$$A = \left[\frac{2\alpha^{\frac{\alpha}{2}}}{\gamma^{1+\frac{\alpha}{2}}\Gamma(\alpha)}\left(\frac{\gamma\beta}{\gamma\beta+\Omega'}\right)^{\beta+\frac{\alpha}{2}}\right]$$

$$a_k = \binom{\beta-1}{k-1}\frac{(\gamma\beta+\Omega')^{1-\frac{k}{2}}}{k-1!}\left(\frac{\Omega'}{\gamma}\right)^{k-1}\left(\frac{\alpha}{\beta}\right)^{\frac{k}{2}} \tag{4}$$

with $\alpha$ being a positive parameter related to the effective number of large-scale cells of the scattering process, and $\beta$ is a natural number where as generalized expression for $\beta$ being a real number can also be derived, with an infinite summation, but it is less interesting due to the high degree of freedom of the proposed distribution. And, the pdf shows very good agreement with the data because of a simple functional form, emphasized by the fact that its $\beta$ parameter being a natural number, which leads to a closed-form representation [11]. Where $\beta$ represents the amount of fading parameter. For simplicity, we have denoted $\gamma = 2(1-\rho)b_0$, finally the parameter $\Omega' = \Omega + 2\rho b_0 + 2\sqrt{2b_0\Omega\rho}\cos(\phi_A - \phi_B)$ represents the average power from the coherent contributions. $\phi_A$ and $\phi_B$ are the deterministic phases of LOS and the coupled-to-LOS scatter components respectively.

Additionally based on $\alpha$ and $\beta$ the flicker factor is defined by [17]

$$S.I. = \left[\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta}\right] \tag{5}$$

## 3.3 pointing errors

In FSO communication systems, the alignment between transmitter and receiver plays a vital role to determine the link performance and reliability. However, misalignment due to sway of buildings by wind loads, thermal expansion and weak earthquakes cause pointing errors and signal fading at the receiver. Pointing errors are denoted by $I_P$, which is discussed below

$$f_{I_p} = \frac{g^2}{A_0^{g^2}}(I_P)^{g^2-1}, 0 \le I_P \le A_0, \tag{6}$$

where $A_0 = [erf(v)]^2$ is the fraction of the collected optical power with $v = \sqrt{\frac{\pi}{2}}\frac{a}{w_z}$, a denotes the radius of the receiver and $w_z$ is the beam width at the distance $L$. The effective beam width is given by the expression $w_{zeq} = \left[\frac{\sqrt{\pi}erf(v)w_z^2}{2ve^{-v^2}}\right]^{\frac{1}{2}}$ where $g_m = \frac{w_{zeq}}{2\sigma}$ is the ratio between the effective beam width and the jitter standard deviation $\sigma_s$.

## 3.4 Combined channel fading models

The combined channel model including atmospheric loss, atmospheric turbulence and pointing errors for $I_{ij}$ is given as [18]

$$f_I(I_i) = \frac{g^2 A I_i^{-1}}{2}\sum_{K=1}^{\beta}\left(a_k\left[\frac{1}{B}\right]^{\frac{\alpha+K}{2}} G_{1,3}^{3,0}\left(\frac{I}{BA_0I_l}\middle|\begin{matrix}1+g^2\\ g^2,\alpha,k\end{matrix}\right)\right) \tag{7}$$

where $B = \left(\frac{\Omega^1+\gamma\beta}{\alpha\beta}\right)$, $G_{p,q}^{m,n}[\cdot]$ is the Meijer's G-function.

# 4 Average BER

## 4.1 BER analysis

We observe bit error rate of individual relay, the influence of SNR on BER analysis is pivotal which is considered here. We consider optical beam after photo-detector of the designed free space coherent optical communication system, the system noise can be described by [17]

$$N_0 = 2q\rho\left(P_0 + P_L\right)R^2 + 4K_b T_{sys} R \tag{8}$$

where $q$ is the electric charge, $\rho$ is the detector efficiency, $K_b$ is the Boltzmann constant, $T_{sys}$ is the system relative Kelvin temperature, $R$ is equivalent load resistance. The signal-to-noise ratio can be expressed as [17]

$$SNR = \frac{4R^2 P_0 P_L h^2}{2q\rho\left(P_0 + P_L\right)R^2 + 4K_b T_{sys} R}, \tag{9}$$

the system symbol error rate considered here is given below [17]

$$P_s = \lambda Q\left(\sqrt{\overline{\gamma} h^2 \frac{3TB}{N(M-1)}}\right) \tag{10}$$

$Q$ function can be expressed in terms of error function using [17], and $T$ represents the symbol time period, $B$ represents bandwidth of OFDM signal, $N$ is the number of sub-carriers, M represents QAM mapping coefficient. We use 32, 64, 128 and 256 QAM in the proposed work. The average BER can be obtained as below.

$$P_e = \int_0^\infty P_s f_{I_i}(I_i) dI_i \tag{11}$$

using $erfc\left(\sqrt{x}\right) = \frac{1}{\sqrt{\Pi}} G_{1,2}^{2,0}\left[x \left| \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix} \right.\right]$ and by substituting eqs. (10) and (7) into eq. (11) we get as

$$P_e = \int_0^\infty \frac{1}{2\sqrt{\Pi}} G_{1,2}^{2,0}\left[\frac{SNRh^2 3TB}{2N(M-1)} \left| \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix} \right.\right] \frac{g^2 A I_i^{-1}}{2} \sum_{K=1}^{\beta} \left(a_{k_m}\left[\frac{1}{B_m}\right]^{\frac{\alpha_m+K}{2}} G_{1,3}^{3,0}\left(\frac{I_m}{B_m A_0 I_l} \left| \begin{matrix} 1+g_m^2 \\ g^2, \alpha_m, k \end{matrix} \right.\right)\right) dI_i \tag{12}$$

by using eq. (14) in [23] the equation can be simplified as below

$$P_e = \frac{g^2 A 2^{\alpha-5}}{\Pi^{\frac{3}{2}}}\left[\frac{1}{B}\right]^{\frac{\alpha}{2}} \sum_{K=1}^{\beta}\left(a_k\left[\frac{1}{B_m}\right]^{\frac{K}{2}} 2^K G_{7,4}^{2,6}\left(\frac{24TB\overline{SNR}}{N(M-1)[BA_0]^2 I_l} \left| \begin{matrix} 1, \frac{1-g^2}{2}, \frac{2-g^2}{2}, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-k}{2}, \frac{2-k}{2} \\ 0, \frac{1}{2}, \frac{-g^2}{2}, \frac{1-g^2}{2} \end{matrix} \right.\right)\right) \tag{13}$$

For the proposed system we assumed multi-hop system with DR relay mode, and number of hops assumed is $L$, hence the average BER is expressed as [17]

$$BER_{AV} = \sum_{j=1}^{L}\left[P_e \prod_{K=i+1}^{L}\left(1 - 2P_e\right)\right] \tag{14}$$

By substituting eq. (13) in eq. (14) we have

$$BER_{AV} \sum_{j=1}^{L}\left[\frac{g^2 A 2^{\alpha-5}}{\Pi^{\frac{3}{2}}}\left[\frac{1}{B}\right]^{\frac{\alpha}{2}} \sum_{K=1}^{\beta}\left(a_k\left[\frac{1}{B_m}\right]^{\frac{K}{2}}\right.\right. \tag{15}$$

$$\times 2^K G_{7,4}^{2,6}\left(\frac{24TB\overline{SNR}}{N(M-1)[BA_0]^2 I_l} \left| \begin{matrix} Y \\ Z \end{matrix} \right.\right)$$

$$\prod_{K=j+1}^{L}\left(1 - \sum_{k=1}^{\beta} \frac{g^2 A 2^{\alpha-4}}{\Pi^{\frac{3}{2}}}\left[\frac{1}{B}\right]^{\frac{\alpha}{2}}\left[\frac{1}{B}\right]^{\frac{\alpha}{2}} \sum_{K=1}^{\beta}\left(a_k\left[\frac{1}{B_m}\right]^{\frac{K}{2}}\right.\right.$$

$$\times 2^K G_{7,4}^{2,6}\left(\frac{24TB\overline{SNR}}{N(M-1)[BA_0]^2 I_l} \left| \begin{matrix} Y \\ Z \end{matrix} \right.\right)$$

where

$$Y = \left[\frac{1-g^2}{2}, \frac{2-g^2}{2}, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-k}{2}, \frac{2-k}{2}, 1\right]$$
$$Z = \left[0, \frac{1}{2}, \frac{-g^2}{2}, \frac{1-g^2}{2}\right]$$

# 5 Outage probability

Outage probability is pivotal in judging the efficiency of the multi-hop FSO communication system, which is defined as the probability that the end-to-end SNR falls below a specified threshold $SNR_{th}$, That is the cumulative density function [CDF] with $SNR_i$ in the $SNR_{th}$, in the form of integral, then outage probability of the system can be expresses as a function of channel combined factor $I_i$ in integral form [17]

$$p_{out,i} = p_r(SNR_i \le SNR_{th}) = \int_0^{SNR_i(I_i)} f\left(I_i\right) dI_i \tag{16}$$

The pdf for the M distribution model from eq. (7) is substituted to eq. (16) we get

$$p_{out,i} = \int_0^{\frac{\rho p_0 h^2 T}{2qBN}} \frac{g^2 A I^{-1}{}_i}{2} \sum_{K=1}^{\beta} \left( a_k \left[\frac{1}{B}\right]^{\frac{\alpha+K}{2}} G_{1,3}^{3,0}\left( \frac{I_i}{BA_0 I_l} \middle| \begin{matrix} 1+g^2 \\ g^2, \alpha, k \end{matrix} \right)\right) dI_i \tag{17}$$

By using eq. (28) from [23] eq. (17) integration term can be expressed as Meijiers G-function as and CDF derived is novel and unique from all the existing literature considering M-distribution channel model

$$p_{out,i} = \frac{g^2 A}{2} \sum_{K=1}^{\beta} \left( a_k \left[\frac{1}{B}\right]^{\frac{\alpha+K}{2}} G_{2,4}^{3,1}\left( \frac{1}{BA_0 I_l} \frac{\rho p_0 h^2 T}{2qBN} \middle| \begin{matrix} 1, 1+g^2 \\ g^2, \alpha, k, 0 \end{matrix} \right)\right) \tag{18}$$

Let $X = \sqrt{\frac{2qB\mu}{\rho P_0}}$ for the normalized threshold value, then the closed form expression for outage probability on single hop is given as

$$p_{out,i} = \frac{g^2 A}{2} \sum_{K=1}^{\beta} \left( a_k \left[\frac{1}{B}\right]^{\frac{\alpha+K}{2}} G_{2,4}^{3,1}\left( \frac{X\sqrt{\frac{N}{\mu T}}}{BA_0 I_l} \middle| \begin{matrix} 1, 1+g^2 \\ g^2, \alpha, k, 0 \end{matrix} \right)\right) \tag{19}$$

As we consider relays in our work the outage probability for multi-hop coherant OFDM system with DF mode, outage probability of multi-hop system dependent on the outage probability of every point to point communication link, it is the probability of the minimum signal-to-noise ratio ($SNR_i$) is lower than the threshold value ($SNR_{th}$) of a target SNR in all relay links in a multi-hop system [17].

$$p_{out}(SNR_{th}) = p_r[\min(SNR_i) \le SNR_{th}]$$
$$p_{out}(SNR_{th}) = 1 - \prod_{i=1}^{L} (1 - p_{out,i}(SNR)_{th}) \tag{20}$$

Therefore substituting eq. (19) into eq. (20), the outage probability of a multihop system can be calculated as:

$$p_{out} = 1 - \prod_{i=1}^{L} (1 - \frac{g^2 A}{2} \sum_{K=1}^{\beta} \left( a_k \left[\frac{1}{B}\right]^{\frac{\alpha+K}{2}} G_{2,4}^{3,1}\left( \frac{X\sqrt{\frac{N}{\mu T}} SNR_{th}}{BA_0 I_l} \middle| \begin{matrix} 1, 1+g^2 \\ g^2, \alpha, k, 0 \end{matrix} \right)\right) \tag{21}$$

## 6 Numerical results

In the work presented eqs. (15) and (21) achieve BER and outage probability of the FSO link over an M channel including path loss, atmospheric turbulence and pointing errors. Both BER and outage probability performance for strong atmospheric turbulence ($\alpha = 1; \beta = 2; C_n^2 = 21,013m^-(2/3)$) and weak atmospheric turbulence ($\alpha = 4; \beta = 9; C_n^2 = 61,014m^-(2/3)$) have been scrutinized and plotted. Statistics available from the plots have been discussed further for reliable system design. We have assumed output power $P_0$ = 70 mW, detector responsivity = 0.5 A/W and the load resistor = 50 Ω.

Figure 3 establishes and analyses the relationship between Average BER and SNR including all the atmospheric effects in eq. (1) and also by varying hop numbers [$L$=1,2,3]. From the plot we can extract that as S.I. increases BER decreases, for example S.I=2 ($\alpha = 1; \beta = 2$) the obtained BER is $10^2$ including two relays [$L = 2$] but at the same time for S.I. = 0.32 ($L = 4$) BER is $10^4$ significant improvement of 20 dB. Also we can notice that by including extra relay node is redundant. So we can conclude that S.I. is inversely proportional to BER, hence need to have low S.I for efficient and reliable FSO system.

Figure 4 establishes the relationship between Average BER and SNR including all the atmospheric effects in eq. (1) additionally varying modulation order of QAM [32,64,128,256]. From the plot we can extract that as order of QAM increases BER increases, for example S.I. = 2.32 QAM ($\alpha = 1; \beta = 2$) the obtained BER is $10^1$ but at the same time for S.I. = 0.40 ($\alpha = 4; \beta = 9$) BER is $10^3$ significant improvement of 20 dB. So we can conclude that S.I is

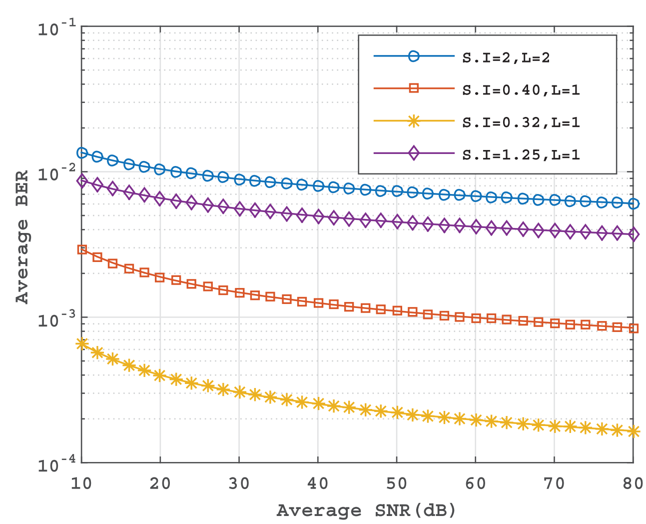


**Figure 3:** BER performance versus SNR for different S.I. and relays.

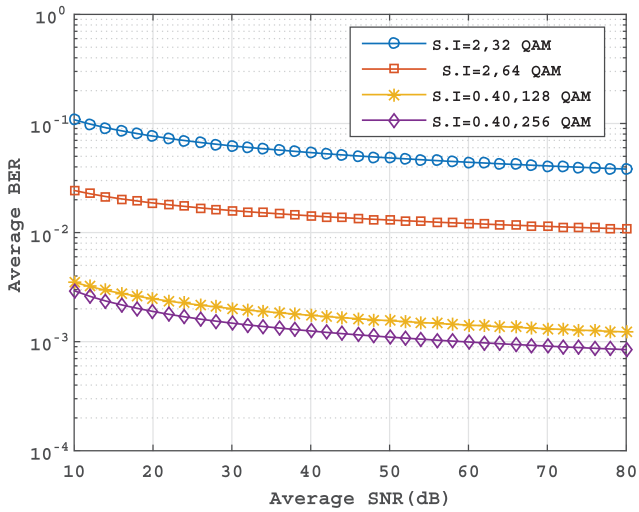


**Figure 4:** BER performance versus SNR for varying QAM modulation order.

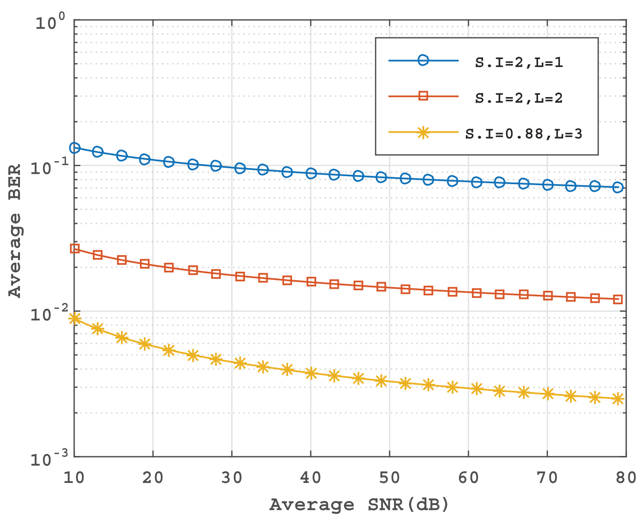


**Figure 5:** BER performance versus SNR for different number of relays.

directly proportional to BER, hence we need to have high order of modulation for efficient and reliable FSO system.

Figure 5 establishes the relationship between Average BER and SNR including all the atmospheric effects in eq. (1) additionally varying the number of hops in multi hop FSO system [$L$=1,2,3]. Its pivotal in present work because it justifies the usage of DF relays in FSO system. From the plot we can infer that as the number of relays increases so as the BER. For single relay BER obtained is $10^1$ as compared to three relays we get $10^3$. So relays plays an important role in accruing the performance of FSO system, which we can convey by using the Figure 5.

In Figure 6 we plotted BER versus SNR under strong turbulence condition for different weather conditions. It is determined from the figure that the value of BER is minimal under very clear weather condition and increases relatively with the presence of haze and fog. For example when we consider SNR equals 80 dB BER, under very clear air and light fog is $10^5$ and $10^2$ respectively. Which is nearly 30 dB improvement in BER.

In Figure 7 we plotted BER versus SNR under weak turbulence condition for different weather conditions. It is determined from the figure that the value of BER is minimal under very clear weather condition and increases relatively with the presence of haze and fog. For example when we consider SNR equals 80 dB BER, under very clear air and light fog is $10^6$ and $10^3$ respectively. Which is nearly 30 dB improvement in BER. In Figure 8 we plotted outage probability versus SNR. It can be noted that as number of relay hops increases outage probability also increases.

As we see in Figure 9a, 9b and 9c BER performance of relay aided system including pointing errors is analysed with single hop [$L = 1$], dual [$L = 2$] and four hop [$L = 4$] respectively with the help of eq. (15) derived, additionally the two factors that determine pointing errors [$I_p$] namely the beam width [$W_Z$] and jitter error [$\delta_s$] used in eq. (6) has been considered in the above figures, as we can see the $X$ axis is the normalized beam width [$W_Z/r$] varied from 1 to 4, and the $Y$ axis is the normalized jitter error [$\delta_s/r$] varied from 0 to 2 by assuming average SNR equals to 50 dB. We can infer from the graph that with the increase of the $W_Z$, the system error rate is reduced. With the increase of the [$\delta_s$], the system error symbol rate increases, which is inevitable in the normalized beam width at low values.

In Figure 9 we can infer that as the number of hops in serial relaying increases then BER also increases. Since we assume SNR equals to 50 dB a constant value the pointing errors and hence system performance completely depends on the addition of relays nodes. And we varied relay hops [$L = 1, 2, 4$] in our work for normalized beam width [$W_Z/r$] equals to 1 and the normalized jitter error [$\delta_s/r$] equals to 2 the BER we obtain are 0.38, 0.42 and 0.58 respectively. Which is an significant increase in BER.

## 7 Conclusion

In this work, the performance of the multi-hop FSO system over a unified distribution model that is Malaga distribution model considering path loss, pointing errors, and atmospheric turbulence have been studied. Closed form mathematical expressions for Average BER and outage probability has been derived considering serial DF relay with OFDM modulation. In addition BER for different weather conditions, pointing errors, turbulence strengths

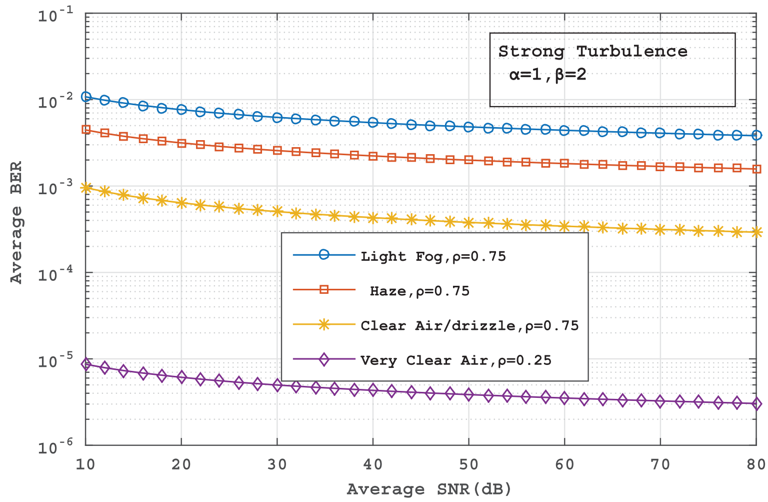


**Figure 6:** BER versus SNR for various weather conditions for the strong turbulence condition.

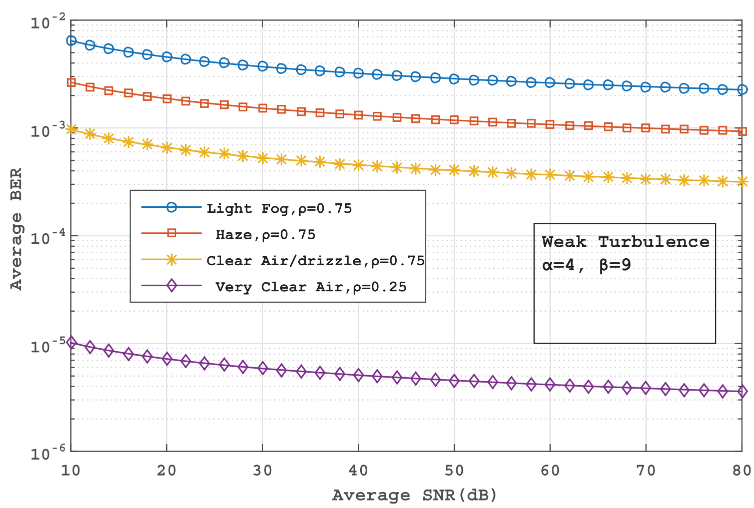


**Figure 7:** BER performance versus SNR for different weather conditions for the weak turbulence.

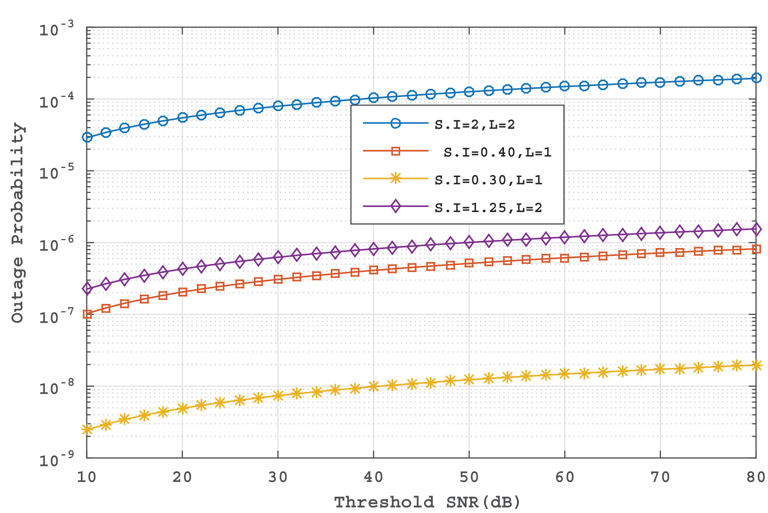


**Figure 8:** Outage analysis versus threshold SNR by varying both S.I. and relays.

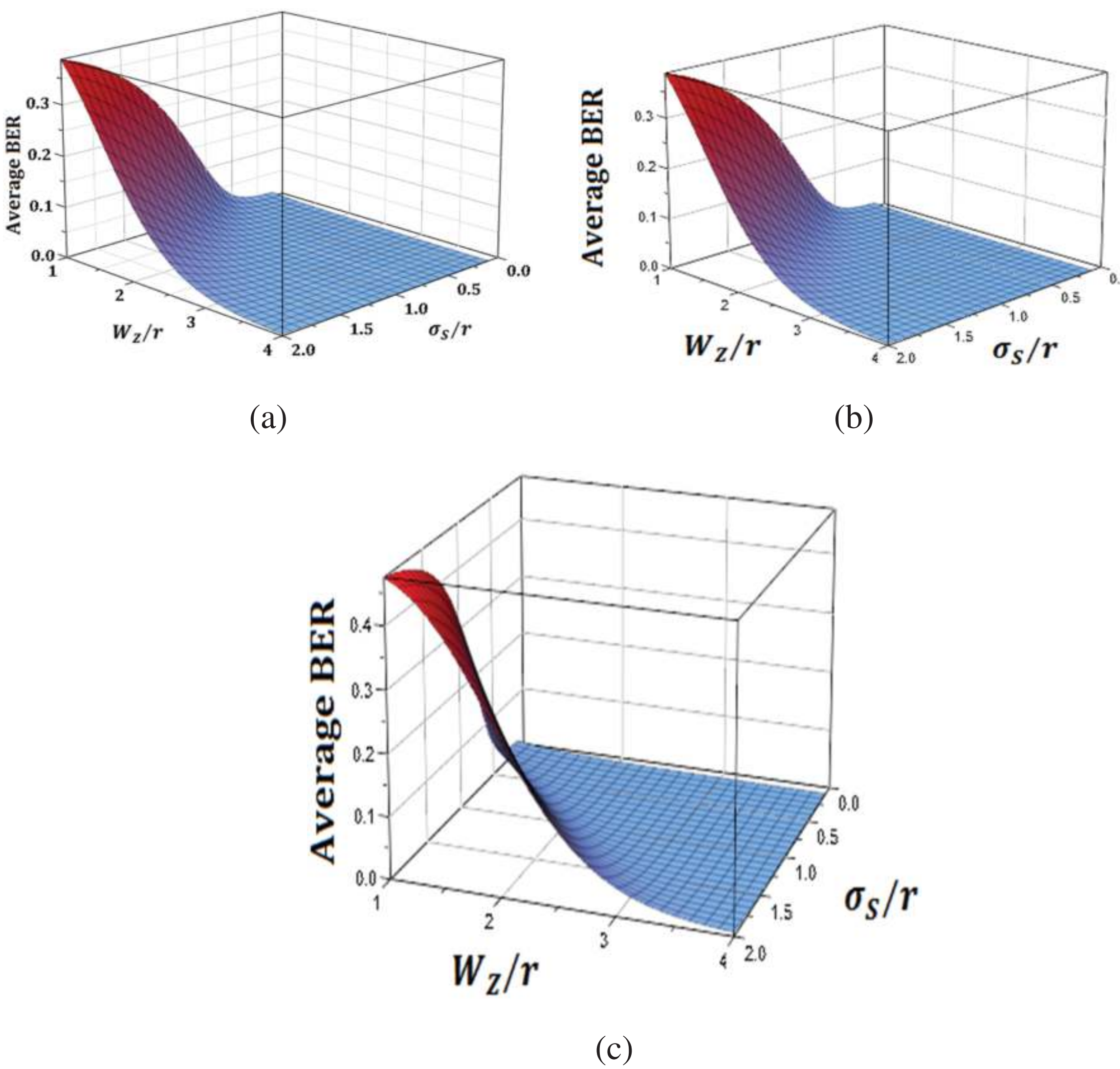


**Figure 9:** BER versus $w_Z/r$ and $\sigma_S/r$.

and relay hops [$L$] has been plotted with respect to SNR for both strong and weak atmospheric turbulence regimes. Hence we can conclude that with multi-hop configuration coverage area and signal power improves which bolsters the usage of relay in FSO communication for efficient and reliable system design.

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