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Peristaltic transport of a viscous fluid in a porous channel with suction and injection

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KEYWORDS

Peristaltic transport;
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Abstract The Peristaltic transport of a viscous fluid in a channel with suction and injection is investigated in the present work. The mathematical modeling has been carried out under long wavelength and low Reynolds number approximation. The analytical solution for velocity field pressure gradient, frictional force and stream function in the wave frame of reference is obtained. The pressure rise and frictional force over one wavelength are obtained. The effect of different parameters on pumping characteristics and frictional forces is discussed graphically. It is observed that pressure rise decreases with increasing permeability parameter and increases with increasing amplitude. It is also observed that for various values of suction parameter k, the pumping curves coincide at a point in the first quadrant due to the suction or injection in the channel. The frictional forces illustrate the opposite behavior compared to pressure rise. The trapping phenomenon for different parameters is presented graphically.

1. Introduction

The study of peristaltic pumping has received considerable attention for the past few decades because of its importance in both biological and mechanical situations. Peristalsis consists of narrowing and transverse shortening of a portion of the tube which then relaxes, while the lower portion becomes shortened and narrowed. Some Bio medical instruments are manufactured based on the principles of peristaltic pumping. A detailed review on peristalsis was presented by Jaffrin and Shapiro [1]. Tang and Fung [2] investigated longitudinal dispersion of particles in the blood flowing in a pulmonary alveolar sheet. Sreenadh and Arunachalam [3] have studied Couette flow between two permeable beds with suction and injection. The analysis given for a single fluid was extended by Brasseur et al. [4] for a two fluid model in a channel. Mansutti et al. [5] have discussed steady flows of non-Newtonian fluids past a porous plate with suction or injection. Chandra and Prasad [6] studied pulsatile flow in circular tubes of varying cross section with suction/injection. A thesis on the Peristaltic pumping in a channel with flexible porous wall has been presented by Reese [7]. Rao and Usha [8] have given

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Nomenclature

a	half-width of the channel	ψ	steam function
b	amplitude of Peristaltic wave	α	permeability parameter
λ	wavelength of the peristaltic wave	m	slip parameter
c	wave speed	k	suction parameter
u, v	velocity components in laboratory frame	$\overline{Q}$	average non-dimensional volume flow rate
p	pressure	q	total flux
ϕ	amplitude ratio	ρ	density
μ	viscosity	V_0	velocity
Re	Reynolds number		

detailed study on Peristaltic transport of two immiscible viscous fluids in a Circular tube. Pontrelli and Bhatnagar [9] studied the flow of a viscoelastic fluid between two rotating circular cylinders subject to suction or injection. Usha et al. [10] investigated Peristaltic transport of two immiscible viscous fluids between two permeable walls.

Majdalani and Zhou [11] have discussed Moderate-to-large injection and suction driven channel flows with expanding or contracting walls. Approximate Analysis of MHD Squeeze Flow between two parallel disks with suction or injection by Homotopy Perturbation method was presented by Domairry and Aziz [12]. Hemadri Reddy et al. [13] have discussed Peristaltic transport of a Jeffrey fluid between porus walls with suction and injection. Several bio fluid flows in psychological systems and blood flow in small blood vessels are reported flow under the mechanism of peristalsis with suction and injection. In view of the several physiological applications it is required to study the Peristaltic transport of a viscous fluid in a channel between porous walls with suction and injection.

In this paper the peristaltic flow of a viscous fluid in a channel with suction and injection is investigated, under long wavelength and low Reynolds number assumptions. The fluid is injected into the channel perpendicular to the lower porous bed with constant velocity V_0 and is sucked out to the upper permeable bed with the same velocity V_0. The velocity, the stream function, the pressure rise and friction force are obtained. The results are deduced and discussed. The trapping phenomenon for different parameters is presented graphically.

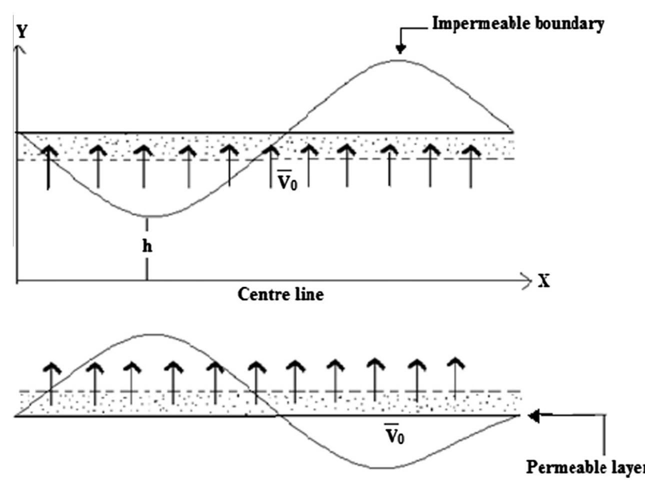

Figure 1 Physical model.

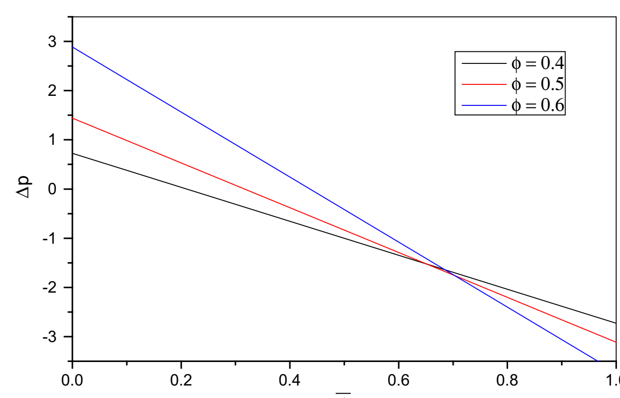

Figure 2 The variation of Δp with $\overline{Q}$ for different values of ϕ with $k = 0.1$ and $\alpha = 0.1$.

2. Mathematical formulation

Consider the peristaltic pumping of a viscous fluid in a porous channel of half width a. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower permeable walls of the channel. The fluid is injected into the channel perpendicular to the lower permeable wall with a constant velocity V_0 and is sucked out of the upper permeable wall with the same velocity V_0. For simplicity, we restrict our discussion to the half width of the channel as shown in Fig. 1. The wall deformation is given by

$$H(X,t) = a + b\sin\frac{2\pi}{\lambda}(X - ct) \tag{1}$$

where b is the amplitude, λ is the wavelength and c is the wave speed.

Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X, Y) .The transformation between these two frames is given by

$$x = X - ct, y = Y, u(x,y) = U(X - ct, Y) - c, v(x,y) = V(X - ct, Y) \tag{2}$$

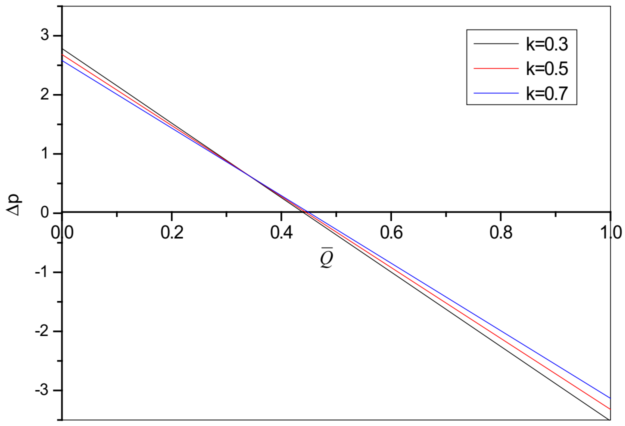

Figure 3 The variation of Δp with $\overline{Q}$ for different values of k with $\alpha = 0.1$ and $\phi = 0.6$.

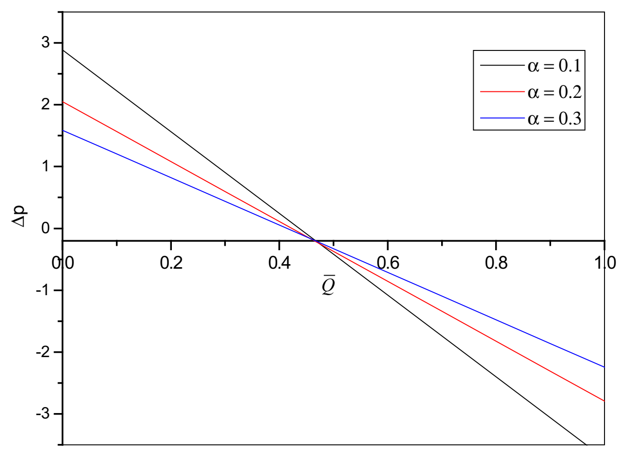

Figure 4 The variation of Δp with $\overline{Q}$ for different values of α with $\phi = 0.6$ and $k = 0.1$.

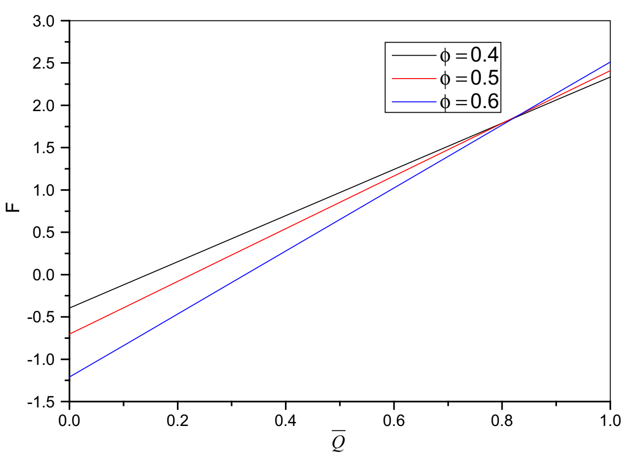

Figure 5 The variation of Frictional force with $\overline{Q}$ for different values of ϕ with $k = 0.1$ and $\alpha = 0.1$.

where U and V are velocity components in the laboratory frame and u, v are velocity components in the wave frame. Further, we assume that the wavelength is infinite. So the flow is Poiseuille type at each local cross section.

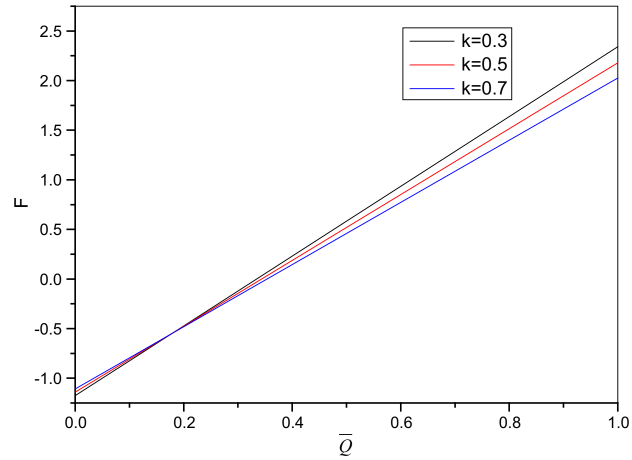

Figure 6 The variation of frictional force with $\overline{Q}$ for different values of k with $\alpha = 0.1$ and $\phi = 0.6$.

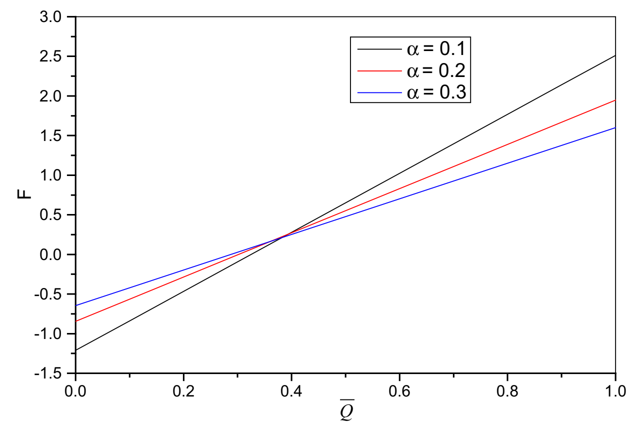

Figure 7 The variation of Frictional force with $\overline{Q}$ for different values of α with $\phi = 0.6$ and $k = 0.1$.

We use the following non-dimensional quantities

$$\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{a}, \ \bar{t} = \frac{ct}{\lambda}, \ \bar{h} = \frac{H}{a}, \ \bar{\psi} = \frac{\psi}{ac}, \ \bar{\phi} = \frac{b}{a},$$
$$\alpha = \left(\frac{am}{\sqrt{k}}\right)^{-1}, \ Re = \left(\frac{ac}{\rho}\right), \sigma = \frac{a}{\sqrt{k}}, \ \bar{p} = \frac{a^2}{\lambda\mu c}p, \quad (3)$$
$$\overline{Q_1} = \frac{Q_1}{c}, \ \overline{v_o} = \frac{v_0}{c}, \ \bar{u} = \frac{u}{c}$$

where R is Reynolds number, ϕ is the amplitude ratio, α is the permeability (including slip) parameter, and k is the suction parameter. The equations governing the motions in non-dimensional form are

$$\frac{\partial^2 u}{\partial y^2} - k\frac{\partial u}{\partial y} = P \quad (4)$$

where $k = ReV_0$, and $P = \frac{\partial p}{\partial x}$

$$Q1 = \frac{P}{\sigma^2} \ \text{(Darcy's law)} \quad (5)$$

The non-dimensional boundary conditions are

$$\psi = 0 \ \text{at} y = 0 \quad (6)$$

$$u = \frac{\partial \psi}{\partial y} = -1 - \alpha\frac{\partial u}{\partial y} \quad \text{at} \quad y = h \quad (7a)$$

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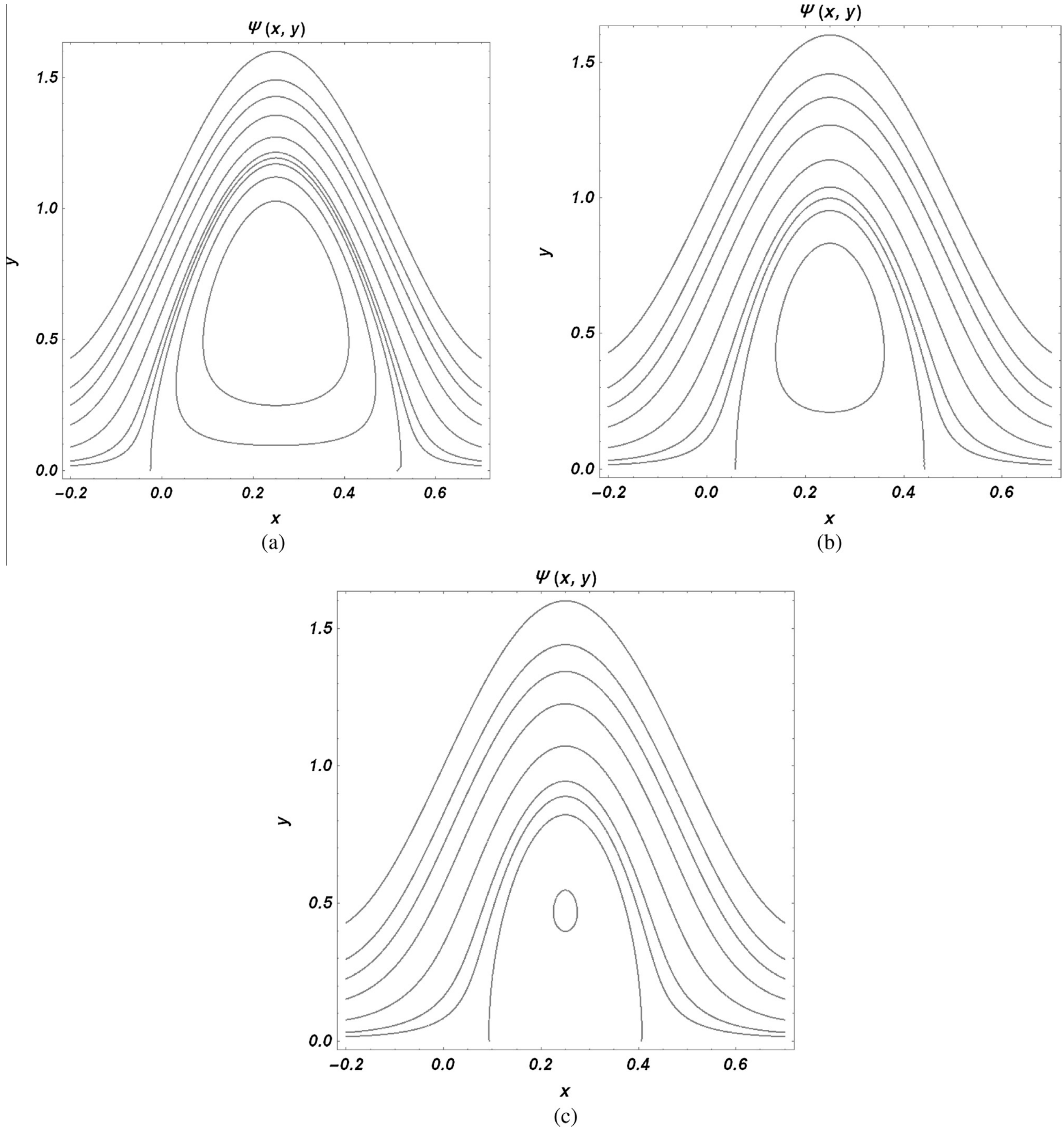


Figure 8 Streamlines for $k = 0.1$, $\phi = 0.6$, $Q = 0.7$ and for different values of α, (a) $\alpha = 0$, (b) $\alpha = 0.2$ and (c) $\alpha = 0.3$.

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{7b}$$

where ψ is the stream function.

3. Solution

Solving Eq. (4) with the boundary conditions (6) and (7), we obtain the velocity as

$$u = -1 + \frac{P}{k^2}\left[(e^{ky} - e^{kh}) + \alpha k(1 - e^{kh}) - k(y - h)\right] \tag{8}$$

Integrating Eq. (8) and using the boundary condition $\psi = 0$ at $y = 0$ we get,

$$\psi = -y + \frac{P}{k^2}\left[\left(\frac{e^{ky}}{k} - \frac{1}{k} - ye^{kh}\right) + \alpha k(1 - e^{kh})y - k\left(\frac{y^2}{2} - hy\right)\right] \tag{9}$$

The volume flux q through each cross section in the wave frame is given by

$$q = \int_0^h u dy = -h + \frac{p}{k^3}\left[e^{kh}(1 - kh) - 1 + \frac{h^2k^2}{2} + h\alpha k^2(1 - e^{kh})\right] \tag{10}$$

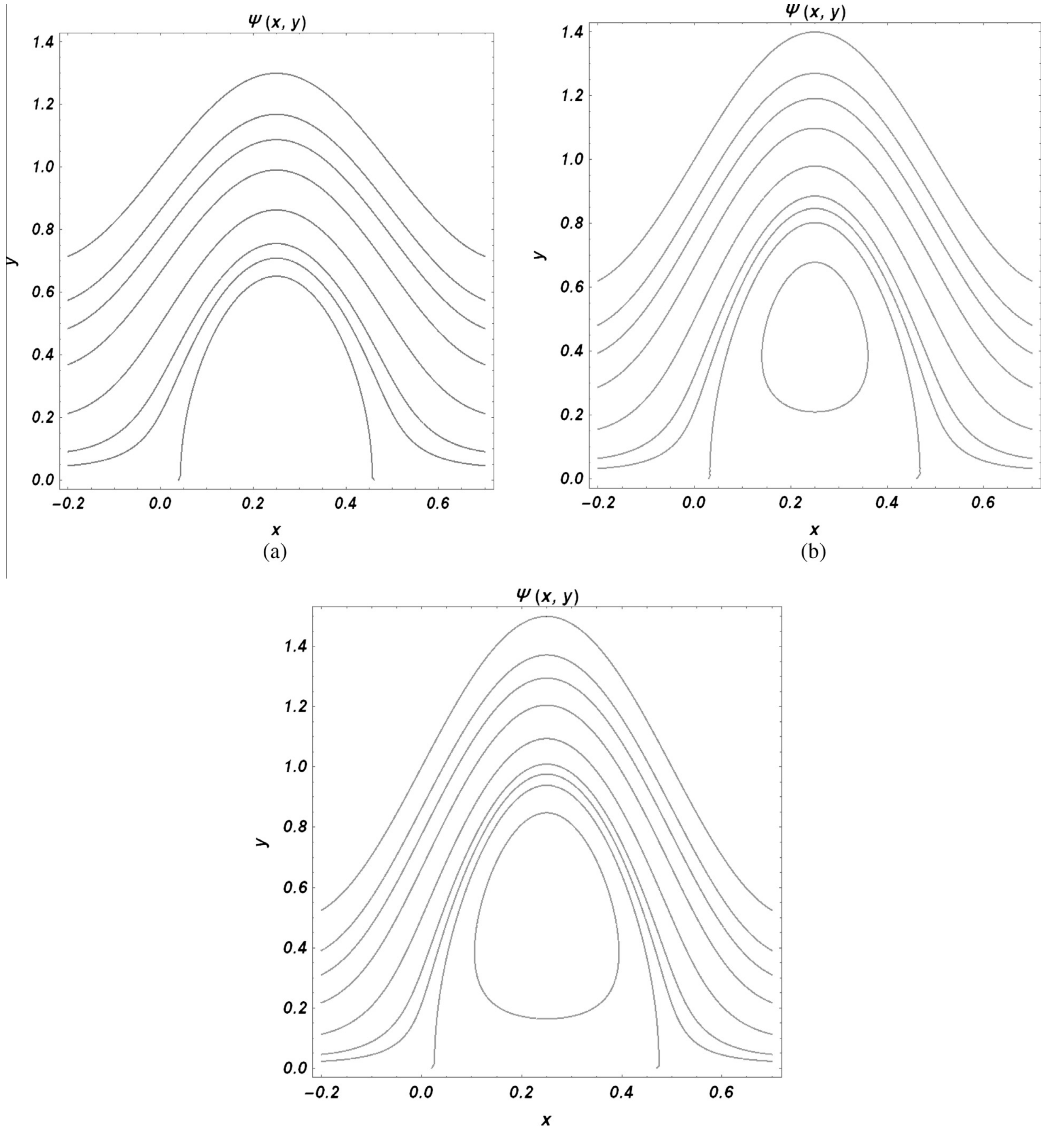


Figure 9 Streamlines for $k = 0.1,\ \alpha = 0.1,\ Q = 0.7$ and for different values of ϕ, (a) $\phi = 0.3$, (b) $\phi = 0.4$ and (c) $\phi = 0.5$.

The instantaneous volume flow rate $Q(X, t)$ in the laboratory frame between the central line and the wall is

$$Q(X,t) = \int_0^H U(X,Y,t)dY = \frac{p}{K^3}\left[e^{kh}(1-kh) + \frac{h^2k^2}{2} + \alpha hk^2(1-e^{kh}) - 1\right] \quad (11)$$

From Eq. (10) we have,

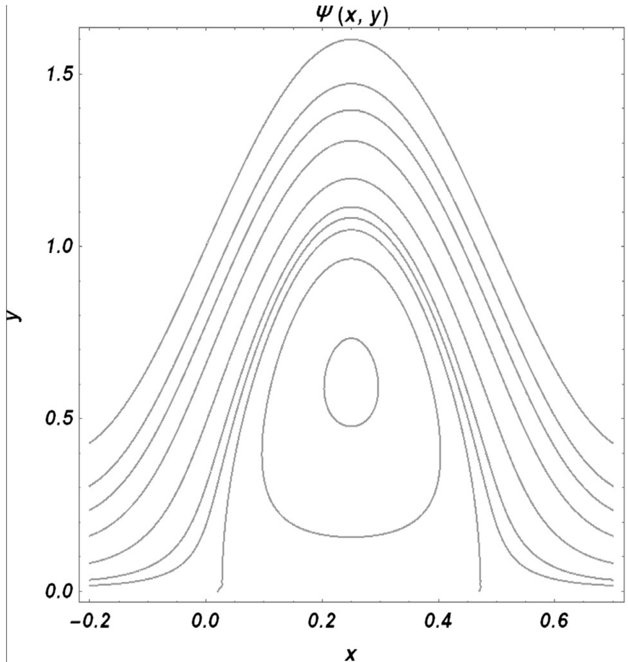
$$\frac{dp}{dx} = \frac{2k^3(q+h)}{2e^{hk}(1-hk) + 2\alpha hk^2(1-e^{hk}) + h^2k^2 - 2} \quad (12)$$

Averaging Eq. (11) over one period yields the time mean flow (time-averaged volume flow rate) $\overline{Q}$ as

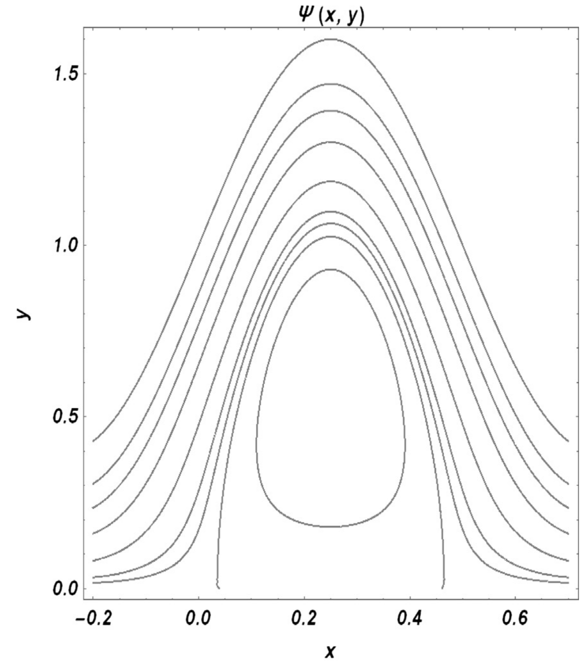
$$\overline{Q} = \frac{1}{T}\int_0^T Q dt = q + 1 \quad (13)$$

4. The pumping characteristics

Integrating Eq. (12) with respect to x over one wavelength, we get the pressure rise (drop) over one cycle of the wave as



(a)



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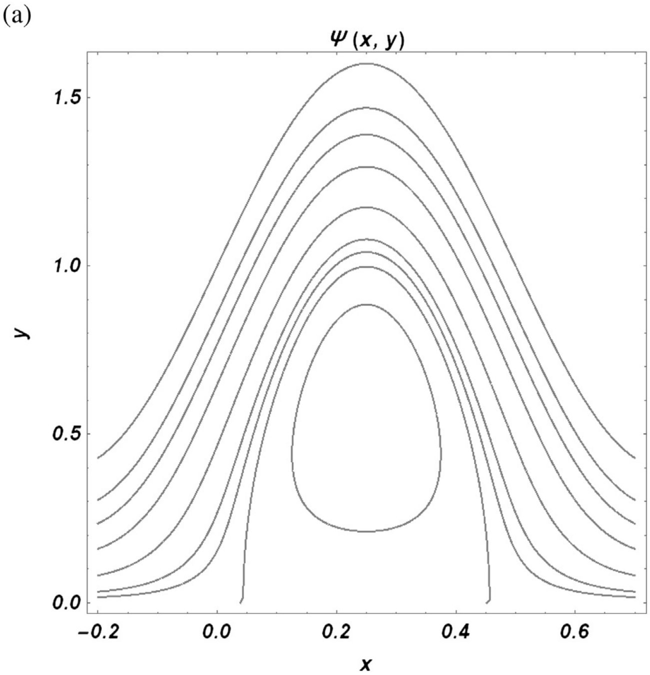

(c)

Figure 10 Streamlines for $\alpha = 0.1,\ \phi = 0.6,\ Q = 0.7$ and for different values of k, (a) $k = 0.3$, (b) $k = 0.5$ and (c) $k = 0.7$.

$$\Delta p = \int_0^1 \frac{2k^3(q+h)}{2e^{hk}(1-hk) + 2\alpha hk^2(1-e^{hk}) + h^2k^2 - 2}dx \quad (14)$$

The time averaged flux at zero pressure rise is denoted by $\overline{Q_0}$ and the pressure rise required to produce zero flow rate is denoted by Δp_0

The dimensionless friction force F at the wall across one wavelength is given by

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right)dx$$

$$\int_0^1 \frac{-2hk^3(q+h)}{2e^{hk}(1-hk) + 2\alpha hk^2(1-e^{hk}) + h^2k^2 - 2}dx \quad (15)$$

5. Results and discussion

5.1. Pressure rise

The variation of pressure rise with time averaged flow rate is calculated from Eq. (14) for different amplitude ratios ϕ and is shown in Fig. 2 for fixed $k = 0.1$ and $\alpha = 0.1$. It is observed that for fixed Δp, the flux increases with increasing ϕ. For a given flux, the pressure rise increases with increasing ϕ.

From Eq. (14) we have calculated the pressure rise with time-averaged flow rate for a different values of k and is shown in Fig. 3 for fixed $\alpha = 0.1$ and $\phi = 0.6$. It is observed that for various values of k the pumping curves coincide at a point in the first quadrant at $\overline{Q} \cong 0.5$. This is due to the suction/injection in the channel. For $\overline{Q} < 0.5$, we observe that the pressure rise decreases with increasing the suction parameter k. For $\overline{Q} > 0.5$, we observe that the pressure rise increases with increasing the suction parameter k.

From Eq. (14) we have calculated the pressure difference as a function of $\overline{Q}$ for different values of α for fixed $\phi = 0.6$, $k = 0.1$ and is depicted in Fig. 4. It is observed that for larger the α the pressure rise decreases in free pumping region whereas the behavior is opposite in co-pumping region.

5.2. Friction force

From Eq. (15), frictional force F is calculated and from Figs. 5–7, it is observed that the frictional force shows opposite behavior compared to the pressure rise.

5.3. Streamlines

Another interesting phenomenon in peristaltic motion is trapping. The streamlines for different values of k, α and ϕ are discussed from Figs. 8–10. In Fig. 8 it is noticed that with increase in permeability parameter α, the size of the trapping bolus decreases. Fig. 9 displays the influence of amplitude on the trapping bolus which increases with increasing amplitude. Fig. 10 reveals that the bolus decreases with increasing suction parameter k.

6. Conclusions

The Peristaltic transport of a viscous fluid in a channel with suction and injection has been studied in the present work under the assumption of long wavelength and low Reynolds number approximation. The expressions for velocity field, stream function, pressure rise and frictional force are determined. It is observed that increase in the suction/injection parameter k, decreases the pressure rise. Increase in the amplitude ratio ϕ, increases the pressure rise. Increase in the permeability parameter α, decreases the pressure rise and the frictional force shows opposite behavior to that of pressure rise with variations k, ϕ and α.

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