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Some analytical calculations on the effect of turbulence on the settling and growth of cloud droplets

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Abstract. Recent research on the interaction of inertial particles with turbulent vortices show that inertial bias causes particles to settle faster in turbulence than in still air as a consequence of particle accumulation in downward fluid velocity regions [*Hainaux et al.*, 2000; *Davila and Hunt* 2001; *Fevrier et al.*, 2000]. To date, these effects have not been included in models of droplet collection growth. In addition earlier models have also neglected the dependence of the collection growth on the radius of the smaller collected drops assuming that the collected drops are stationary. We have considered all these effects and have still been able to solve the collection equation analytically and the earlier *Baker*[1993] results are retrievable as a special case when these effects are turned off. The results presented in this study are in closed form and are therefore extremely attractive for precipitation parametrizations in large scale models.

Introduction

Since the publication of the paper by *Maxey* [1987] where it was shown that inertial particles because of a bias in their trajectories towards regions of low vorticity, settle at rates different from their terminal velocities, there has been a lot of theoretical and experimental work on particle behaviour in turbulence. Numerical studies (e.g. [*Fevrier et al.*, 2000]) have shown that inertial bias cause particles to accumulate in the shape of twisted tube-like structures into fluid velocity areas of larger time coherence and under gravity settle faster than in still fluid as a consequence of particle accumulation in downward fluid velocity regions. The increase of settling velocity was described by *Wang and Maxey* [1993] as the preferential 'sweeping' due to the inertial bias and the fact that particles approach the turbulent structures of the flow usually from above. *Wang and Maxey*[1993], using a DNS of 3D homogeneous turbulence, found increases as high as 50% of the mean settling velocity compared to the terminal velocity. The concept of an increase in the swept volume due to turbulence was taken further by *Pinsky et al.*[2000] and they also gave estimates of turbulence induced increases in the collection efficiencies and concluded that for a 20 μm collector drop the mean collision efficiencies are greater than the conventional values by a factor ranging from 1.2 to 6. In another recent paper *Shaw* [2000] hypothesized that bursts of high supersaturation are produced in turbulent convective clouds through interactions between cloud droplets and the small scale structure of atmospheric turbulence associated with vortex tubes with diameters of the order of 10^{-2} to 10^{-1} m i.e between the Kolmogorov and Taylor microscales. The most definitive summary to date on the effect of turbulence on all aspects of cloud microphysics are by *Jonas* [1996] and *Vaillancourt and Yau* [2000].

In a recent paper by *Davila and Hunt*[2001] which has been also confirmed by *Fevrier et al.* [2000], it was shown that under certain conditions inertial particles can settle upto 80% faster in turbulence. From the *Davila and Hunt* [2001] paper it is found that if the ratio of the terminal velocity V_s to the maximum velocity in the vortex is less than about 1.0, the effect of the vortex on the settling velocity is determined by the non-dimensional ratio F_p of the stopping (or starting) distance of the droplet to the characteristic radius of the trajectory of the droplet around the vortex (when $F_p \leq 1$). In effect F_p is a particle Froude number or a re-scaled Stokes number and is expressed as $F_p = V_s{}^2\tau_p/\Gamma$ where τ_p is the particle relaxation time ($\tau_p = (2/9)(\rho_p/\rho_a)a^2/\nu$, with ν the kinematic viscosity of the fluid ($\sim 10^{-5}m^2s^{-1}$ in air and ρ_p and ρ_a are the densities of the particle and the fluid) and Γ the circulation of the vortex ($\Gamma(k) \sim \epsilon^{1/3}k^{-4/3}$). When the effect of the inertia of a particle on its trajectory is very small, it passes round the vortex and the net change in the actual velocity V_t from its value V_s in still fluid is negligible. However when the inertia is significant the particles are flung outwards and under these conditions the largest velocity amplifications $\sim$ 100% occur. For very large inertia the vortex has negligible effect on the settling rate since the particles crash through the vortex core. We find that the velocity amplification ratio expressed in terms of the drift integral D and the effec-

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tive volume fraction $\alpha \sim a^{-4}$ is given by $\sim (1-\alpha D)$. D is essentially the drift integral of the length scale corresponding to the difference between the vertical settling distance with and without the vortex, and the tendency of the settling droplets with small inertia to be swept into the down-flow side of the vortex causes D to be negative and the settling velocity to be greater than the terminal velocity when $F_p < 1$. These calculations depend sensitively on the droplet size. For example, if one considers the droplet radii between 5-20 μm as in this study applicable to cloud droplets, then F_p can vary between 1.84×10^{-5} and 0.075; the Stokes number S_t between 0.05-0.74; the radius of the droplet trajectory around the vortex which is given by the ratio of Γ and the still air fall velocity V_s varies between 0.05 - 0.003 m. Under these conditions D becomes more negative with decreasing values of V_s which varies as a^2 for small cloud droplets. Thus, for the parameters relevant to this study the velocity amplification effect fades with increasing drop radii and becomes vanishingly small for $a \sim 40\mu m$. The theoretical models by *Davila and Hunt* [2001] and by *Fevrier et al.* [2000] have also been corroborated by laboratory experiments by Srdic and Fernando [*Srdic*, 1998] using the Digimage system. In this paper we shall directly use the *Davila and Hunt* [2001] results to compute the exact fall velocity enhancements for cloud droplets with radii $\sim 10\mu m$. But first we shall show that for typical cloud parameters it is indeed the microscale vortices that cause the maximum velocity amplification : the speeding up effect of the vortices is a maximum when $F_p \sim 1$ or when $V_s^2 \tau_p = \Gamma$. Typically cloud turbulence intensities $\sim 100 cm^2 s^{-3}$ and the small droplet radii $\sim 10\mu m$ and from the definitions of Γ and τ_p it is easy to work out that the eddy length scale $\sim 10^{-3} m$ i.e. the microscale eddies.

Our calculations show that the dependence of $(V_t - V_s)/V_s$) (i.e. the difference in the actual fall velocity in turbulence from the still air values normalised over the still air fall velocity) on the droplet radii is an exponentially decaying function so that for droplets with radii $\sim 10\mu m$ this can be upto 80% while for 20 μm droplets this falls to $\sim$ 20%. Recent results by *Pinsky et al.* [2000] have shown higher turbulence induced velocity increases -for example for sheared horizontal flows their results show that the turbulence induced realtive velocity difference is of the same order (effectively an increase $\sim$ 100%) as in the difference in fall velocities. In addition they also suggest that the collision efficiencies can be several times larger leading to a very large overall effect. A curve fitting shows that for our model, the size dependence of the amplification effect can be expressed as $(V_t - V_s)/V_s = \alpha e^{-\beta a}$ with α=5.5 and β=0.173μm^{-1}. The maximum error that can arise from our using this expression with the specified values of α and β instead of the exact Davila and Hunt calculations $\sim$ 5%. Although turbulence can significantly increase collision efficiencies, we have deliberately refrained from including this effect in order to single out the fall velocity amplification effects. In this study the calculations based on a single cloud droplet interacting with a single vortex with radii $\sim 1mm$ are appropriate for small cloud droplets with radii $\sim 10\mu m$ interacting with the microscale eddies in clouds. Since cloud droplets upto and below this size range are usually the most numerous, the cumulative effect over a spectrum of droplet sizes interacting with microscale eddies with a spectrum of vortex radii should be significant. More details on these aspects will be dealt with in a later paper where we shall also consider the effects of more energetic eddies in addition to the microscale ones.

Impact of turbulence induced terminal velocity modifications on drizzle evolution

In this section we consider the evolution of drizzle as a result of collision and coalescence.

The rate of change of drop radius R due to collisions with droplets of radius r can be expressed as :

$$4\pi R^2 \frac{dR}{dt} = \frac{\pi}{\rho_l}(R+r)^2 |V(R) - V(r)| q_l \rho_a E(r,R) \quad (1)$$

where E(r,R) is the collection efficiency of droplets of radius r by drops of radius R, q_l is the cloud water mixing ratio in drops of radius r, ρ_a and ρ_l are the air and liquid densities. When the larger drops have radii $R > 30\mu m$ then the fall speed V(R) can be expressed as $V(R) = JR$ where J =8630 s^{-1} [*Johnson*, 1982]. This expression for spherical droplets is applicable in the intermediate size range, i.e between the Stokes' law region and the square root law (i.e $30\mu m < R < 0.6mm$) regime. If the smaller droplets of radius r that are captured have radii $< 30\mu m$ then their fall velocities can be expressed as $V(r) = k_0 r^2$ where $k_0 = 1.2 \times 10^8 m^{-1} s^{-1}$. This quadratic dependence of the settling velocity is valid for spheres in the Stokes' regime and can be easily applied to cloud droplets up to $30\mu m$ radius [*Rogers and Yau*, 1994]. It must be noted that these standard expressions for the drop velocities widely used in cloud microphysical calculations apply to different regimes and are therefore not continuous at the transition regime. However, we are concerned in this analysis with droplet pairs of unequal sizes and therefore the discontinuity at the transition regime is easily overcome in our subsequent analysis.

From the results of the preceding section when one considers the effect of turbulence then the modified velocities of the smaller droplets of radii r which are likely to be the most affected are expressed as $V(r) = k_0 r^2 (1 + \alpha e^{-\beta r})$.

Let us now consider how the drop collection growth is affected when we include turbulence induced fall velocity modifications and we shall only consider the case of the larger drops of radii $R > 30\mu m$ capturing smaller droplets of radii $r < 30\mu m$ and we shall as-

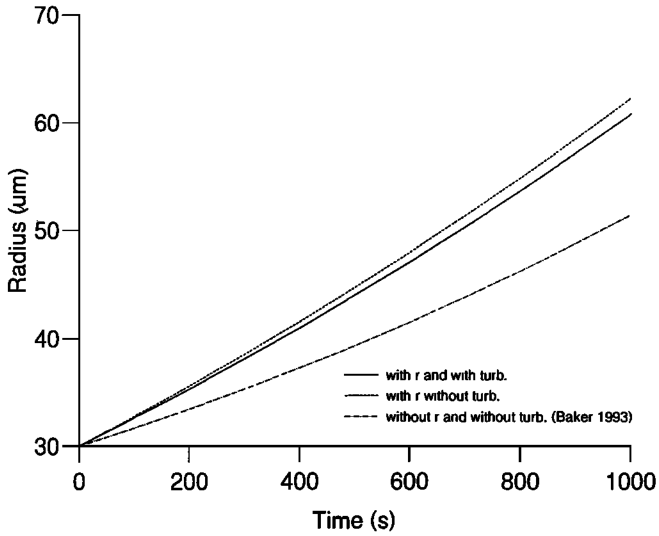

Figure 1. Droplet collection growth calculated from Equation (2) including the dependence on r and also with and without turbulence effects. The original Baker (1993) results where these effects are not considered are also shown.

sume that only the small droplets are affected by turbulence. Equation (1) reduces to $4\pi R^2 \frac{dR}{dt} = \frac{\pi \rho_a}{\rho_l}(R+r)^2[JR - k_0 r^2(1+\alpha e^{-\beta r})]q_l E$ which approximates to $\frac{dR}{dt} = \frac{J\rho_a}{4\rho_l}(1+\frac{2r}{R})[R - J^{-1}k_0 r^2(1+\alpha e^{-\beta r})]q_l E$.

Since r $\sim 10\mu m$ and R $\geq 30\mu m$, $r^2/R^2 \sim 0.09$ or smaller and can be neglected. Similarly, by neglecting terms $\sim r^3$ and retaining the most dominant terms in the above equation we can simplify it as $\frac{dR}{dt} = A_1 J[R + 2r - J^{-1}k_0 r^2(1+\alpha e^{-\beta r})]$ where $A_1 = \frac{q_l E \rho_a}{4\rho_l}$ and as a result an exact analytic integration yields :

$$R = R_0 e^{JA_1 t} + [2r - J^{-1}k_0 r^2(1+\alpha e^{-\beta r})]JA_1 t$$

$$= R_0 e^{JA_1 t} + mJA_1 t \qquad (2)$$

where R_0 is the value of R at t=0 and $m = [2r - J^{-1}k_0 r^2(1+\alpha e^{-\beta r})]$.

For values of J, A_1 relevant to many clouds one can express the exponential term as $e^{JA_1 t} \approx 1 + JA_1 t$. Then we can re-write the equation (2) as $R = R_0(1 + \frac{R_0+m}{R_0}JA_1 t) \approx R_0 e^{nJA_1 t}$ where $n = \frac{R_0+m}{R_0} = 1 + 2r/R_0 - \frac{k_0 r^2(1+\alpha e^{-\beta r})}{JR_0}$. When we neglect the turbulence induced velocity modifications then $\alpha \to 0$ and $\beta \to 0$ and further when $r/R_0 \to 0$ then $R = R_0 e^{J\frac{q_l E \rho_a}{4\rho_l}t}$ or, $R = R_0 e^{\frac{t}{\tau_c}}$ which is identical to the equation (9b) in the analysis of *Baker*[1993] with the collection growth time given by $\tau_c = \frac{4\rho_l}{\rho_a q_l JE}$. In contrast, by including the effects of turbulence induced velocity modifications the collection growth time is given by $\tau_c^* = \tau_c(1 + 2r/R_0 - \frac{k_0 r^2(1+\alpha e^{-\beta r})}{JR_0})^{-1}$. From the preceding equation $\tau_c^* < \tau_c$. This includes the dependence on r as well as the effects of the terminal velocity modifications.

In Figure 1 we show (by using equation 2) the growth rate for $q_l = 0.5 gm^{-3}$, E=0.5, $R_0 = 30\mu m$ and $r = 10\mu m$. We find that a 30 μm drop grows to twice that size in about 1000 s whereas in the original *Baker*[1993] formalism the drop would achieve a radius of only about 50 μm in the same time. In the analysis by *Baker*[1993] the dependence on r was ignored which implies that the collector drop captures only stationary droplets which in a turbulent medium is unlikely. From equation (1) it is clear that the rate of change of the large drop radius R depends sensitively on the velocity difference V(R)-V(r). Turbulence effects will tend to increase the values of V(r) causing a smaller relative velocity, which in turn will tend to increase τ_c^* compared to the case when this effect is not considered and this is clear from Figure 1. It is important to note however, that in this analysis we have excluded the effects of amplified collision efficiencies in turbulence. If higher values of collision efficiencies are used then the growth rate will increase in turbulence even if there is a decrease in the relative velocity. It is also possible that situations may arise when droplet collection occurs between droplet pairs whose radii are comparable and are within the 10-30 μm range. In this case it will become necessary to include the turbulence effects on both V(R) and V(r) -in contrast to the case we have just analysed where we did not consider turbulence effects on the larger of the two droplet pairs, since, with respect to the present model, the amplification effect for droplet radii $> 30\mu m$ is very small. For the case where both the radii are between $10 - 30\mu m$ turbulence will increase the fall velocity of both the droplet pairs proportionately and since turbulence effects depend sensitively on the size, the amplification effects will be larger for the smaller of the two droplets.

Having analysed the accretion growth of precipitation particles we can now easily calculate the equilibrium

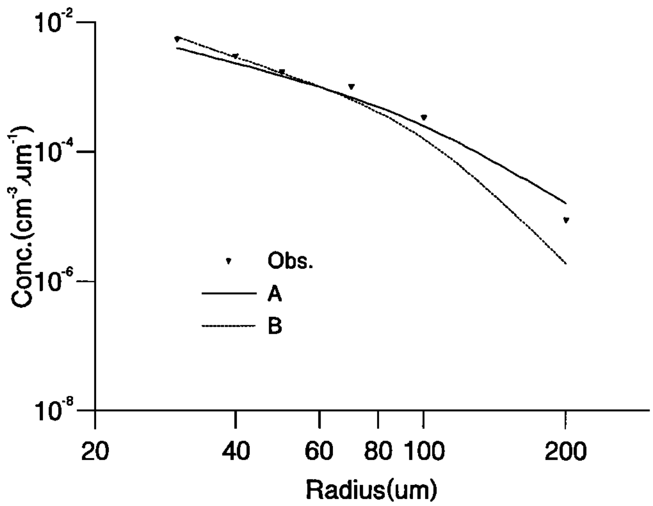

Figure 2. Observed and modelled raindrop spectrum for the FIRE case study. The line marked A refers to the calculations based on Equation (3) which includes the r dependence and the effects of settling velocity modifications induced by turbulence, and the line marked B refers to the case when these are not considered.

raindrop distribution as a function of height z within a spreading plume in manner outlined by *Baker*[1993] and subsequently by *Austin et al.* [1995] and the details of the derivations can be found in both these papers. However, the point to note is that we have a different expression for the calculation of the collection time constant i.e. τ_c^* in place of τ_c. *Austin et al.* [1995] have incorporated turbulence effects in their analytic equilibrium calculations to predict drizzle spectra for a stratocumulus cloud -however, in their calculations they have only considered the effect of including air velocity fluctuations caused by turbulence but have not considered the effect of fall velocity enhancements. In our calculations we shall consider both these effects. The final expression for the equilibrium rain drop distribution is given by

$$\eta(z,R)dR = \frac{Q\tau_c^*}{2R}(erf[-\frac{z-\Delta z(t_*)}{\sqrt{2}\sigma_z(t_*)}] - erf[-\frac{z-\Delta z(t_*)-h}{\sqrt{2}\sigma_z(t_*)}]) \quad (3)$$

where $t_* = \tau_c^* ln(R/R_0)$ and h is the cloud depth and the $\Delta z(t)$ is the distance fallen by the plume center $\Delta z(t) = -\int_0^t JR(t')dt'$ which simplifies to $\Delta z(t) = -\tau_c^* JR_0(exp(t/\tau_c^*) - 1)$.

In Equation(3) Q is the source term given by $Q = \frac{\text{autoconversion rate}}{4\pi R_0^3 \rho_l/3}$ and σ_z the plume standard deviation is calculated from the standard Taylor's theorem (e.g. see equation B4 in *Austin et al.* [1995]). We shall now apply equation (3) to a drizzling stratocumulus case study widely studied during the FIRE programme (First ISSCP (International Satellite Cloud Climatology Program) Regional Experiment) and we shall compare our results for the de-coupled case presented by *Austin et al.*[1995] for the Electra flight 8 for 14 July 1987. The results are shown in Figure2. The details of the observations can be found from *Austin et al.* [1995]. The line marked A refers to our calculations based on equation (3) where the dependence on the radius of the collected drop and the associated the effects of turbulence are considered. For the line marked B these effects are not considered and so corresponds to the original analytical solution proposed by *Baker* [1993]. For all the calculations the auto-conversion term is equal to $7 \times 10^{-9} kgm^{-3}s^{-1}$, σ_w (the in-cloud vertical velocity fluctuation)=0.4 ms^{-1}, T_L(the Lagrangian integral time scale)=450 s. The distribution is calculated at 200 m above the cloud base for a cloud 400 m thick with a lwc of 0.48g/Kg. From this graph also it is clear that our calculations yield a much closer fit to the observations, particularly for the large drop sizes suggesting that when one includes the dependence of the drop collection on the radii of the smaller collected drops and the associated turbulence induced terminal velocity modifications in the equilibrium rain drop spectrum there is a significant increase in the large drop concentrations matching observations from stratocumulus clouds.

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