

PAPER • OPEN ACCESS

Tuning of PID controller using optimization techniques for a MIMO process

To cite this article: S Thulasi dharan *et al* 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **263** 052019

View the [article online](#) for updates and enhancements.

Related content

- [Particle swarm optimization based PID controller tuning for level control of two tank system](#)
Anju K Vincent and Ruban Nersisson
- [PID controller tuning using metaheuristic optimization algorithms for benchmark problems](#)
Vishal Gholap, Chaitali Naik Dessai and V Bagyaveereswaran
- [Comparison of PID and Fuzzy Controller for Position Control of AR.Drone](#)
A Prayitno, V Indrawati and I I Trusulaw

Tuning of PID controller using optimization techniques for a MIMO process

S Thulasi dharan, K Kavyarasan and V Bagyaveereswaran

School of Electrical Engineering, VIT University, Vellore - 632014, Tamil Nadu, India

E-mail: vbagyaveereswaran@vit.ac.in

Abstract: In this paper, two processes were considered one is Quadruple tank process and the other is CSTR (Continuous Stirred Tank Reactor) process. These are majorly used in many industrial applications for various domains, especially, CSTR in chemical plants. At first mathematical model of both the process is to be done followed by linearization of the system due to MIMO process and controllers are the major part to control the whole process to our desired point as per the applications so the tuning of the controller plays a major role among the whole process. For tuning of parameters we use two optimizations techniques like Particle Swarm Optimization, Genetic Algorithm. The above techniques are majorly used in different applications to obtain which gives the best among all, we use these techniques to obtain the best tuned values among many. Finally, we will compare the performance of the each process with both the techniques.

1. Introduction

Many chemical industries process are of dynamic and highly nonlinear due to many process and manipulated variables. This type of MIMO (Multi Input and Multi Output) process are difficult to control. There are many methods to control the MIMO process. In this paper, we proposed the design of feedback control for controlling the levels of a quadruple tank system and also for controlling temperature and concentration of the chemical in a CSTR process. Which has two input and output variables, also known as 2 X 2 MIMO process[2][3]. It is a dynamic system and also with high nonlinearities. So, at first the mathematical model of the four tank system [1], and CSTR system will be done by using the principle of linearization and the proposed system state space model can be represented by using Jacobian matrix.

RGA is a powerful tool that has been employed in the multivariable control to choose controlled variable and manipulated variable pairs that produces desirable response.[2][3]

In highly complex multivariable processes which involves large number of feedback control loops, Decouplers are added in order to reduce interaction between minor process variables[3]. In this prototype two pumps are used in delivering the water to the tanks from the reservoir. Tanks are connected in such a way that change in either of the inlet will have an effect on both the process variables. And also the input flow of the any liquid, the inlet temperature will effect the temperature of the CSTR and also the PID tuning controllers are majorly and commonly used technique for process control in many industries



because they are easy to employ and cost benefit.[10]. Due to its simplicity, it has the capability of providing the satisfactory performances [5]. In addition to that, there are many tuning methods. It also has the additional functions which will increase the performance of the process [7][10][11]. For the tuning of the PID parameters there are many optimization techniques are available like Zeigler Nicholas method, Ant Colony algorithm, Particle Swarm Optimization, Genetic Algorithm. Among them in this project we are going to use two optimizations such as Particle Swarm Optimization (PSO), Genetic Algorithm(GA).

The systems with tuned parameters from the above optimization techniques is given to the controllers, which is designed in the SIMULINK and assess the performance by comparing the response of two methods with two proposed techniques (PSO and GA). This paper comprises of the section-2 deals with the modeling of both the systems, section-2 deals with the tuning of PID with two techniques for both systems, section-3 discusses about parameters tuning, section-4 for simulation and results.

2. Modelling of Systems

2.1 Modelling of Quadruple Tank

In this proposed work one of the process we have taken is quadruple tank system as a process model which is shown in the figure 1. Tank 1 and tank 4 are fed by pump 1, similarly tank 2 and tank 3 are fed from Pump 2. The Pump outlet is split into two halves by simple hand operated valves. The dimensions of the system are given in the table 1.

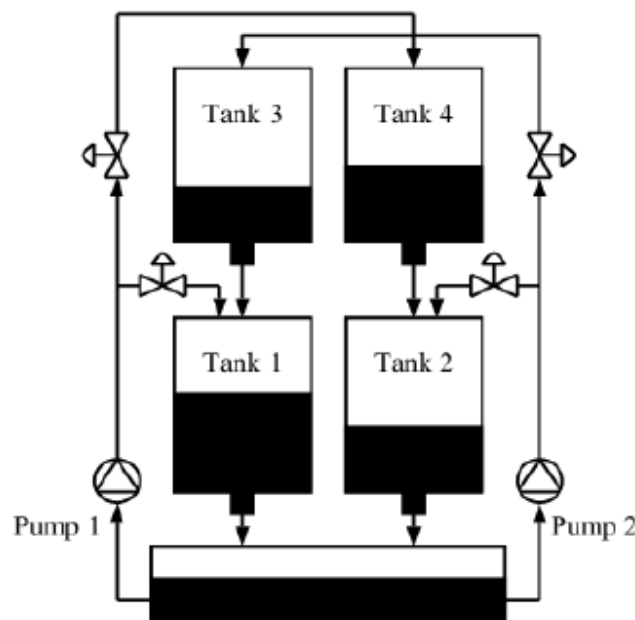


Figure 1. QTP Process diagram

Table 1. Quadruple Tank Process tank dimensions

$A_i(\text{cm}^2)$ -area of tank i	28
$a_i(\text{cm}^2)$ -area of drain in tank i	0.16,0.13,0.16,0.13
γ_i -ratio of flows in the valves	0.5,0.5
k_i -Pump proportionality constant	0.67,0.74
$g (\text{cm}/\text{s}^2)$ – Gravitational constant	9800

The process is modeled by writing mass balance equation for individual tanks by considering both inlets simultaneously.

Thus, mass balance equations for individual tanks are ,

$$A_1 \frac{dh_1}{dt} = -a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + \gamma_1 k_1 v_1 \quad (1)$$

$$A_2 \frac{dh_2}{dt} = -a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + \gamma_2 k_2 v_2 \quad (2)$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2 v_2}{A_3} \quad (3)$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1 v_1}{A_4} \quad (4)$$

Where h_i is the level in tank i .

Representing the above governing equation in state space,

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{a_3}{a_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{a_4}{a_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \mathbf{u} \quad (5)$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \mathbf{x} \quad (6)$$

where, $\mathbf{x} = [h_1 \quad h_2 \quad h_3 \quad h_4]$ and $\mathbf{u} = [v_1 \quad v_2]$

$$T_i = \frac{A_i}{a_i} \sqrt{\frac{2h_i^0}{g}}$$

$$\begin{bmatrix} H_1(s) \\ H_2(s) \end{bmatrix} = G(s) \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1} + D$$

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

(7)

$$G_{11} = \frac{2.57}{62.7s + 1}, G_{12} = \frac{1.5}{(23.8s + 1)(62s + 1)}$$

$$G_{21} = \frac{1.4}{(30s + 1)(90s + 1)}, G_{22} = \frac{2.8}{90s + 1} = \frac{H_2}{U_2}$$

$$H_1 = G_{11}U_1 + G_{12}U_2$$

$$H_2 = G_{21}U_1 + G_{22}U_2$$

Controller-1 forces H_1 towards its set point, U_1 and U_2 also affects H_1 through G_{12} . Similarly Controller-2 adjusts H_2 towards its set point, U_2 and U_1 affects H_2 through G_{21} .

2.2 CSTR Modeling

The CSTR process schematic diagram is in figure 2. The input flow rate and outlet flow rate of a reactant A is F with concentration of C_{Af} at temperature T_f with concentration C_A at temperature T . And also the coolant jacket with input flow rate of coolant at temperature T_j in and T_j out. The dimensions of the system are given in the table 2.

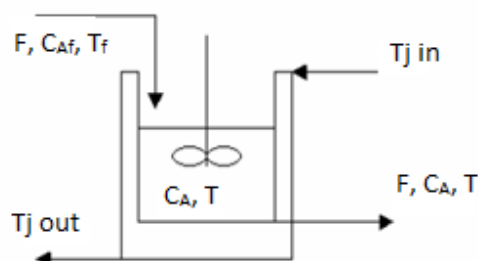


Figure 2. CSTR Process diagram

Table 2. Process parameters of CSTR

Parameters	Values
F/V, hr-1	1
Ko,hr-1	9703x3600
$(-\Delta H), K_{cal}/K_{mol}$	5960
E, K_{cal}/K_{mol}	11843
$\rho C_p, K_{cal}/m^3 \text{ } ^\circ c$	500
Tf, $^\circ c$	25
$C_{af}, K_{cal}/m^3$	10
UA/V, $K_{cal}/m^3 \text{ } ^\circ c, hr$	150
Tj, $^\circ c$	25

Linearization of dynamic equations

$$F1(CA, T) = \frac{dCA}{dt} = \frac{F}{V(CAf - CA)} - Ko \exp\left(-\frac{E}{RT}\right) CA \quad (8)$$

$$F2(CA, T) = \frac{dT}{dt} = \frac{F}{V(Tf - T)} + \left(-\frac{\Delta H}{\rho Cp}\right) Ko \exp\left(-\frac{E}{RT}\right) CA - \frac{UA}{V\rho Cp(T - Tj)} \quad (9)$$

$$X = \begin{bmatrix} CA & -CAS \\ T & -TS \end{bmatrix}$$

$$U = \begin{bmatrix} F & -FS \\ Tj & -TjS \end{bmatrix}$$

Representing the above governing equation in state space matrices as,

$$A = \begin{bmatrix} -\frac{F}{V} - Ks & -CasKs \left(\frac{\Delta E}{RTs^2}\right) \\ \left(-\frac{\Delta H}{\rho Cp}\right) Ks & -\frac{F}{V} - \frac{UA}{V\rho Cp} + \left(-\frac{\Delta H}{\rho Cp}\right) Ks \left(\frac{\Delta E}{RTs}\right) s^2 \end{bmatrix}$$

$$B = \begin{bmatrix} (Caf - Cas)V & 0 \\ (Tf - Ts)V & \frac{UA}{V\rho Cp} \end{bmatrix} C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$G_{11} = \frac{1.4364s + 1.5393}{(s - 0.580)(s - 0.88)} \quad G_{12} = \frac{-0.0249}{(s - 0.580)(s - 0.88)}$$

$$G_{21} = \frac{-13.171s-12.5517}{(s-0.580)(s-0.88)} G_{22} = \frac{0.3s+0.347}{(s-0.580)(s-0.88)}$$

2.3 Selection of Controlled and Manipulated Variables



Figure 3. Schematic process diagram

As seen from the above schematic diagram in figure 3, the process is considered as MIMO model which consists of 2 input variables (U_1 and U_2) and 2 output variables (H_1 and H_2). Change in either of the input variables will affect both output variables. So, totally 4 combinations of controlled and manipulated variable pair is possible ($(U_1$ Vs. H_1 and U_2 Vs. H_2) or $(U_1$ Vs. H_2 and U_2 Vs. H_1)).

But practically it is not possible to design control loops for all the pairings. Relative Gain Array (RGA) is a tool that measures the interaction between the controlled and manipulated variable. It recommends the possible pairing of controlled and manipulated variable which produces optimum results. RGA matrix is,

$$\lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \lambda_{11} = \frac{1}{1 - \frac{K_{21}K_{12}}{K_{22}K_{11}}}$$

$$\lambda_{12} = \lambda_{21} = 1 - \lambda_{11}$$

$$\lambda_{11} = \lambda_{22}$$

Where K_{ij} is the steady state gain of G_{ij} .

For Quadruple Tank system the RGA matrix is given by

$$\lambda = \begin{bmatrix} 1.412 & -0.412 \\ -0.412 & 1.412 \end{bmatrix}$$

For CSTR system the RGA matrix is given by

$$\lambda = \begin{bmatrix} 2.3 & -1.3 \\ -1.3 & 2.3 \end{bmatrix}$$

As seen from the above matrix, $\lambda_{12} < 0$ and $\lambda_{21} < 0$ opening or closing of either of the loops will have an adverse effect on other loop which may produce oscillatory response. So, H_1 should not be paired with U_2 and also H_2 should not be paired with U_1 . Thus H_1-U_1 and H_2-U_2 pairing gives the effective results.

2.4 Decouplers

After selecting major controlled and manipulated variables by inferring RGA matrix, minor interactions between H_2-U_1 and H_1-U_2 are eliminated by adding decouplers in the process.

$$T_{21} = \frac{-G_{21}}{G_{22}}$$

$$T_{12} = \frac{-G_{12}}{G_{11}}$$

For Quadruple tank ,

$$T_{21} = \frac{-0.5}{30s + 1}, T_{12} = \frac{-0.5769}{23s + 1}$$

For CSTR System,

$$T_{12} = \frac{-0.0249}{1.4364s + 1.5393}, T_{21} = \frac{-0.13171s + 12.5517.5769}{23s0.3s + 0.347 + 1}$$

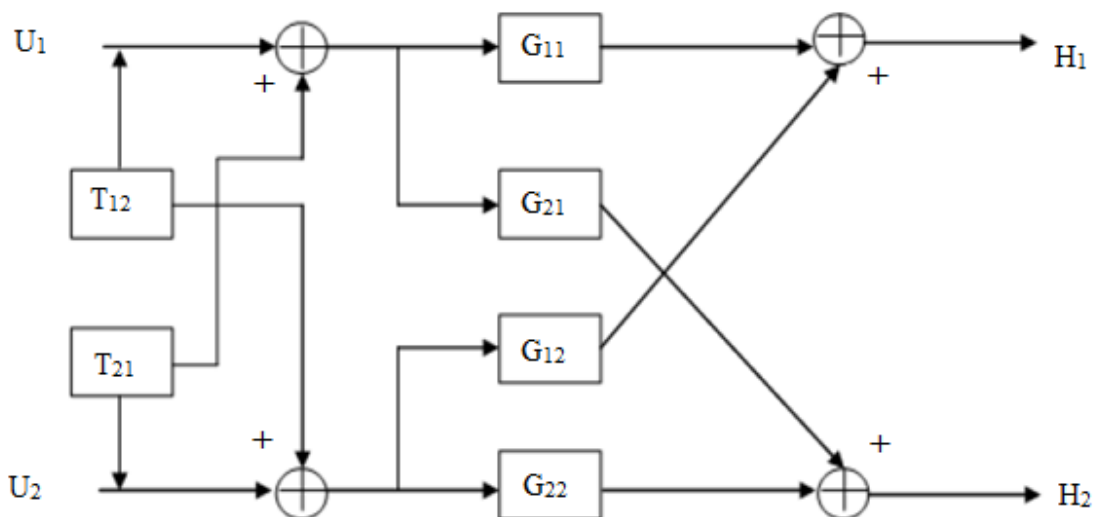


Figure 4. Decouplers

3. Controller Tuning

Tuning of PID controller parameters are made with many methods but for this above two process we have taken two optimization techniques such as Particle swarm Optimization (PSO), Genetic Algorithm (GA)

3.1 Particle Swarm Optimization

This algorithm works with having population (swarm) of candidate solution (particle). Every particle is a candidate solution to optimize the problem. The best solution particles are moved around in the search-space. The movements of the particles are guided by their own best known position in the search-space as well as the entire swarm's best known position. In this technique a set of particles are put in search space with randomly choosing velocity and position. The algorithm for the proposed models is shown in figure 5. And implementing this PSO in the MATLAB using the cost function for the respective models. The tuned PID parameters are tabulated in following tables 3,4 and 7,8 for QTP and CSTR process respectively.

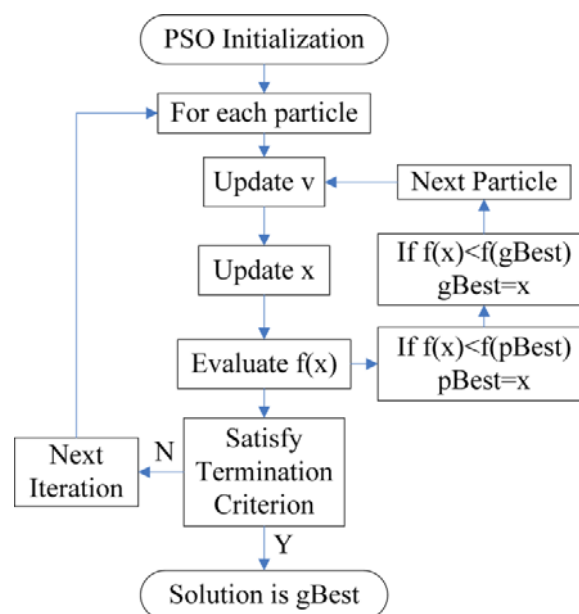


Figure 5. PSO algorithm

Initially we set the values of algorithm constants as :

Inertia weight factor, $W = 0.3$

Acceleration constants, $C_1, C_2 = 1.5$

Run the program with the PSO algorithm with 100 iterations in MATLAB and returned the final optimal fitness function value as “pbest” and global optimum solution as “Gbest”

3.2 Genetic Algorithm

It is a class of evolutionary algorithms, which generate solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover. The

algorithm for the propose models is shown in figure 6. And implementing this GA in the SIMULINK ‘Optimization’ tool box by using the cost function for the respective model. The tuned PID parameters are tabulated in following tables 5, 6 and 9, 10 for QTP and CSTR process respectively.

Iterations (n)=100

Lower Limit of Controller parameters = [0 0 0]

Upper Limit of Controller parameters = [100 100 100]

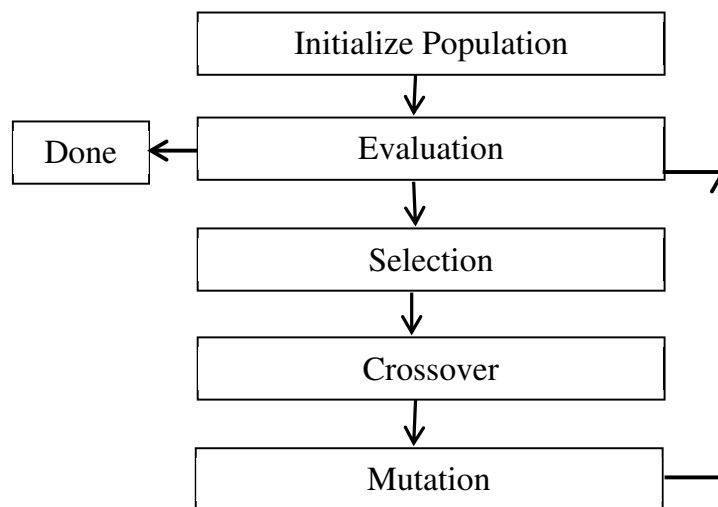


Figure 6. GA algorithm

4. Simulation and Results

After modeling the process, RGA matrix is formed as explained in the section 2 by which control and manipulated variable pairs are chosen. Then the process is splitted into two halves and tuned separately as explained in the section 2.

4.1 Quadruple Tank System Controller Parameters

Tuned PID parameters using PSO algorithm:

Table 3. Controller parameters for Loop 1

Parameters	Tuned values
Kp	39.4728
Ki	20.1375
Kd	13.3591

Table 4. Controller parameters for Loop 2

Parameters	Tuned values
Kp	47.1728
Ki	20.7982
Kd	30.1844

Tuned PID parameters using GA:

Table 5. Controller parameters for Loop 1

Parameters	Tuned values
Kp	93.239
Ki	78.493
Kd	82.478

Table 6. Controller parameters for Loop 1

Parameters	Tuned values
Kp	99.993
Ki	92.734
Kd	27.114

4.2 CSTR System Controller Parameters

Tuned PID parameters using PSO algorithm:

Table 7. Controller parameters for Loop 1

Parameters	Tuned values
Kp	3.8048
Ki	12.2341
Kd	36.27

Table 8. Controller parameters for Loop 2

Parameters	Tuned values
Kp	1.3691
Ki	9.944
Kd	21.5722

Tuned PID parameters using GA:

Table 9. Controller parameters for Loop 1

Parameters	Tuned values
Kp	89.827
Ki	35.76
Kd	0.013

Table 10. Controller parameters for Loop 2

Parameters	Tuned values
Kp	98.65
Ki	38.383
Kd	1.971

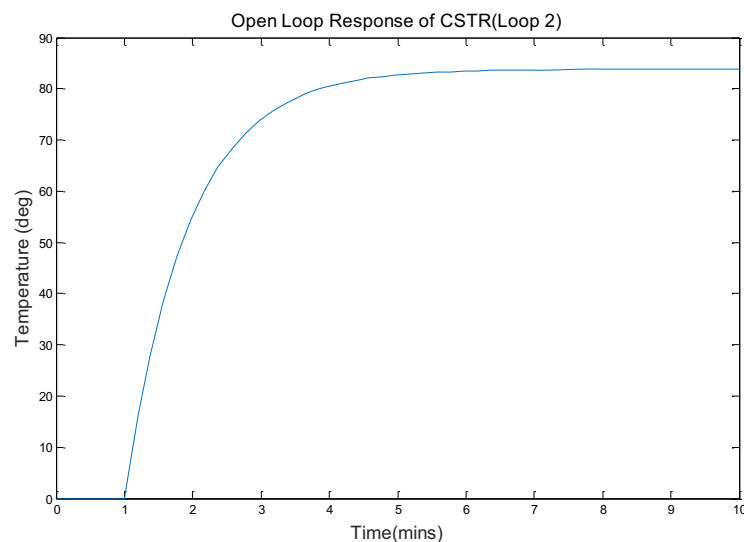


Figure 7. Open loop Response of CSTR for and 2nd loop

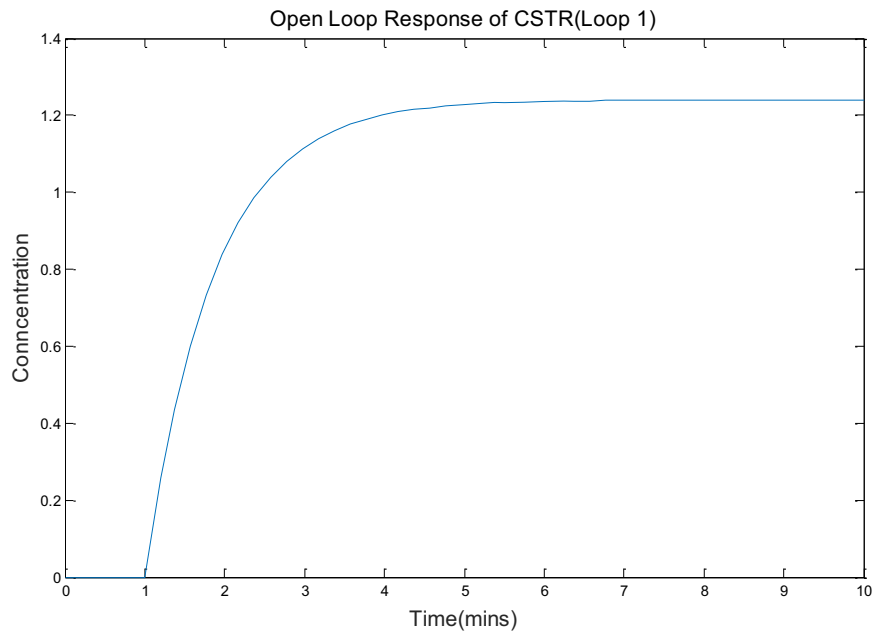


Figure 8. Open loop Response of CSTR for 1st loop

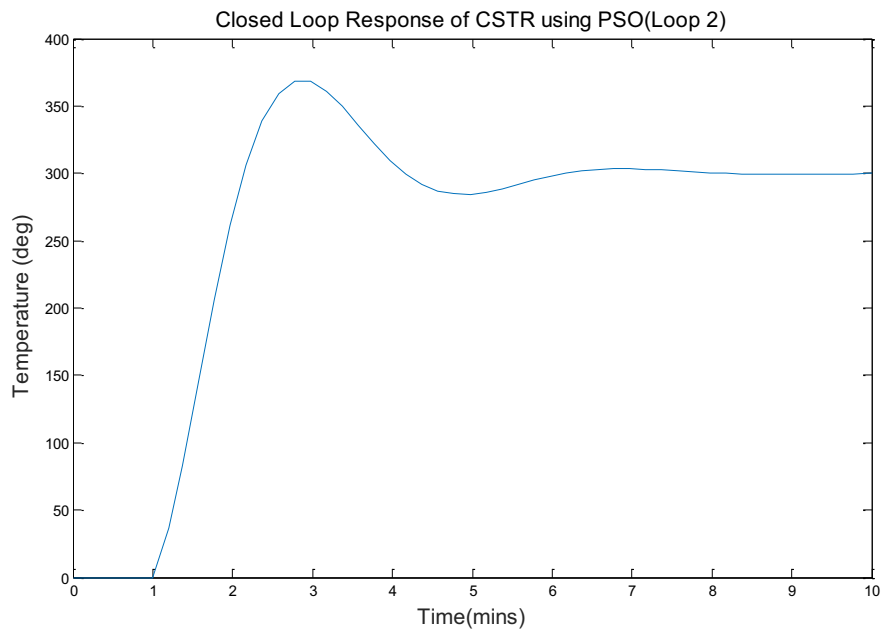


Figure 9. Closed loop Response of CSTR using PSO for 2nd loop

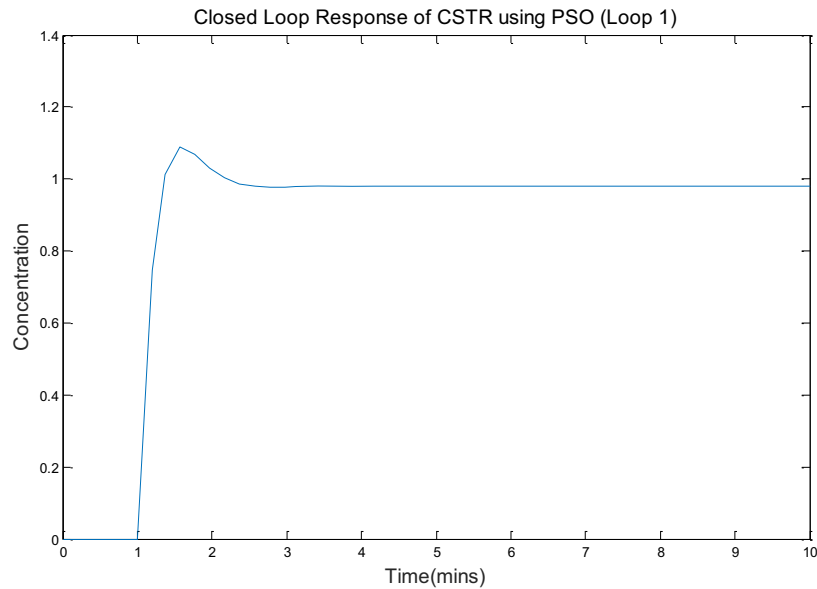


Figure 10. Closed loop Response of CSTR using PSO for 1st loop

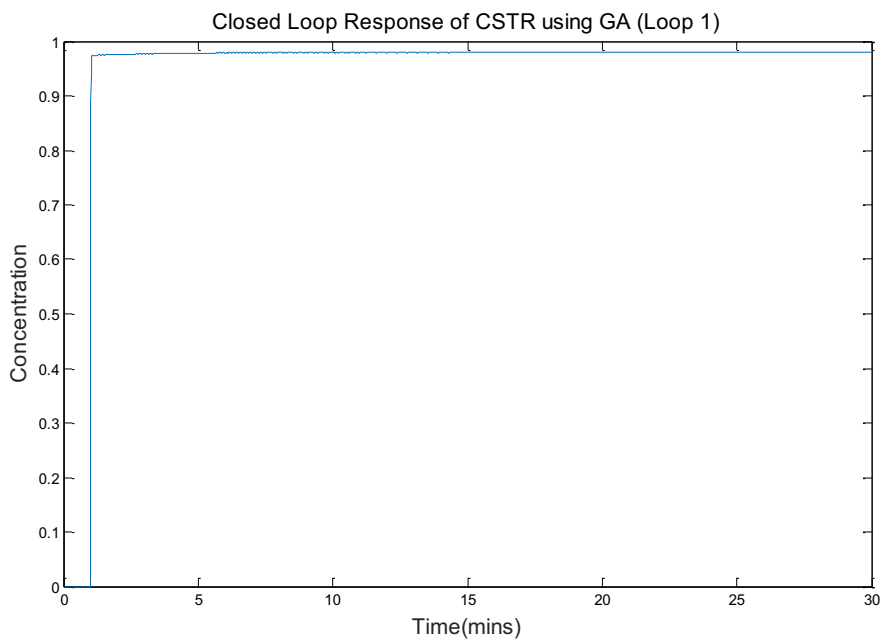


Figure 11. Closed loop Response of CSTR using GA for 1st loop

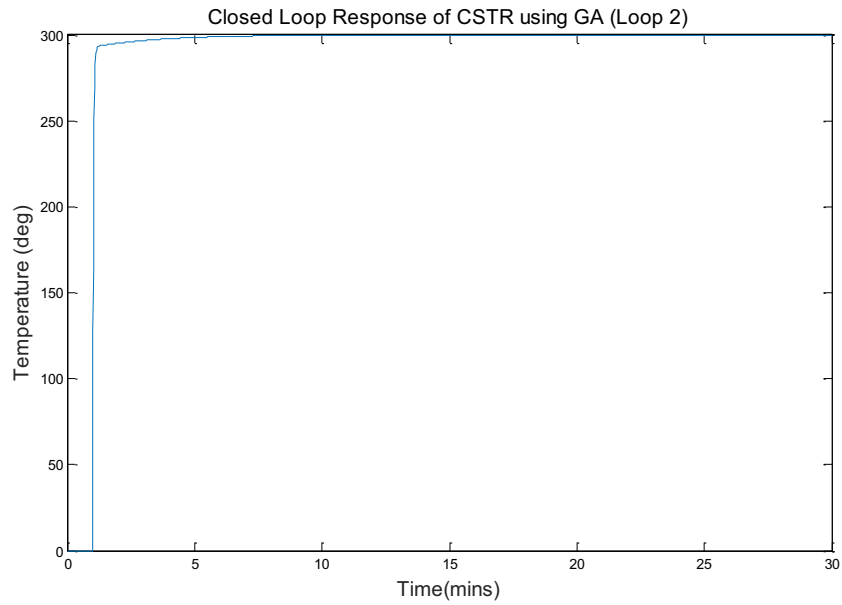


Figure 12. Closed loop Response of CSTR using GA for 2nd loop

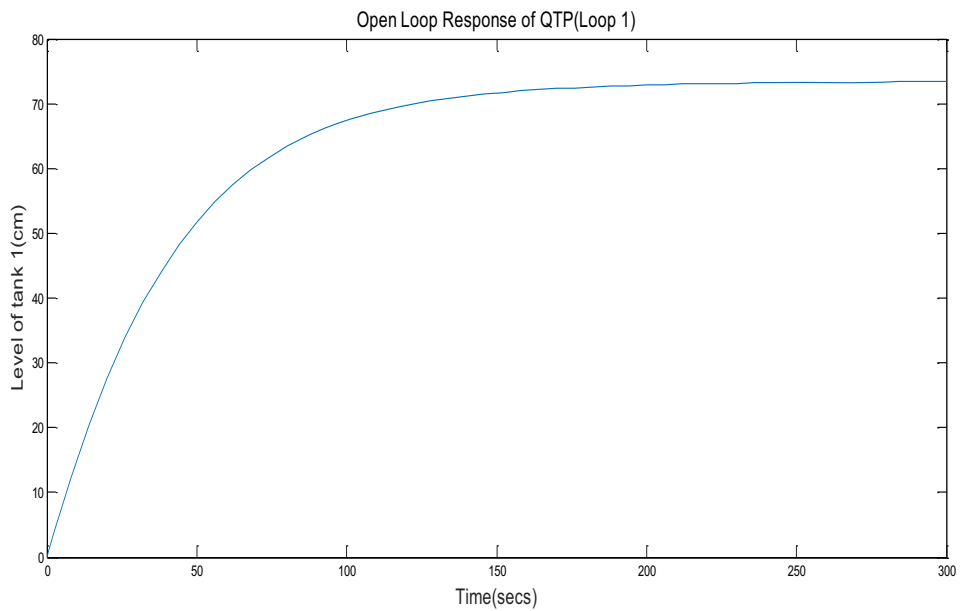


Figure 13. Open Loop Response of QTP 1st Loop

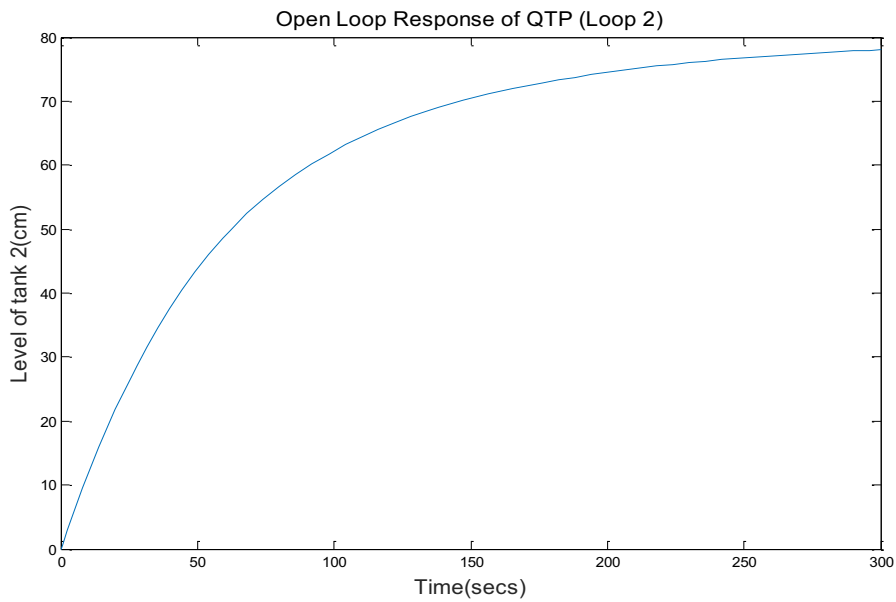


Figure 14. Open Loop Response of QTP 2nd Loop

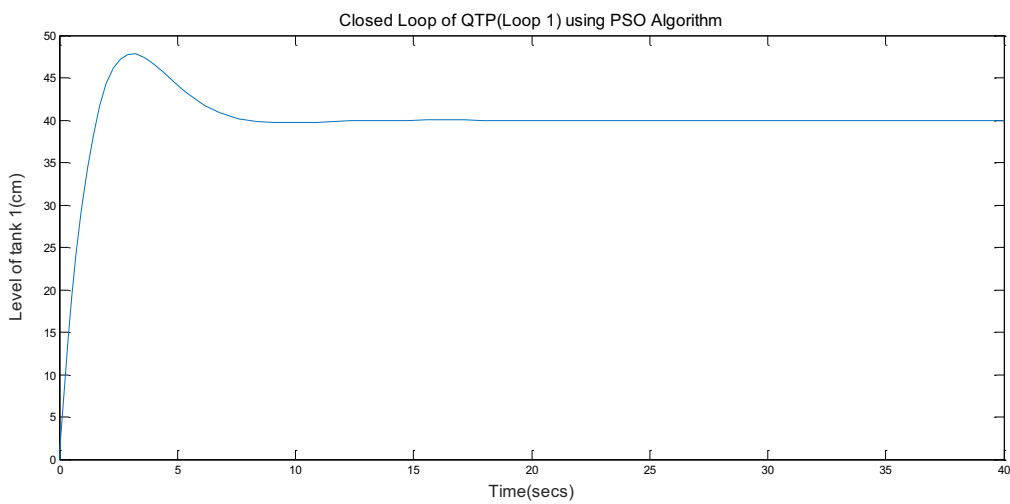


Figure 15. Closed loop Response of QTP using PSO for 1st loop

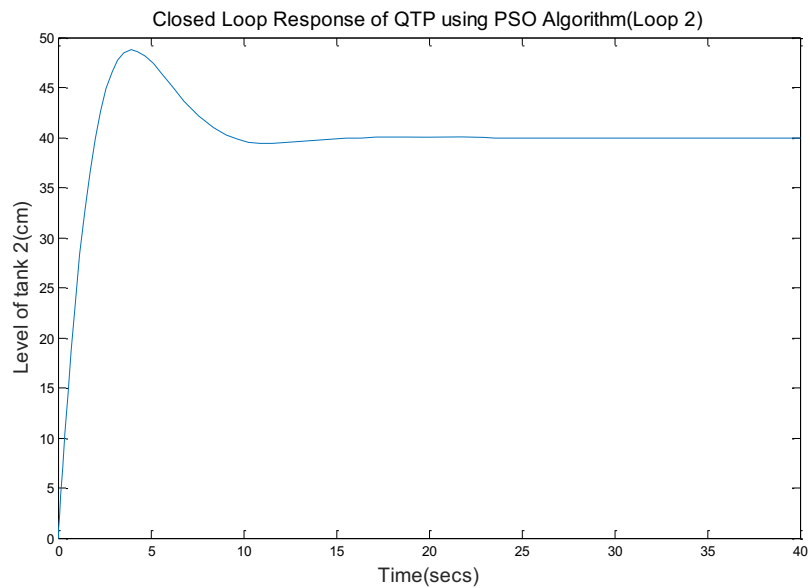


Figure 16. Closed loop Response of QTP using PSO for and 2nd loop

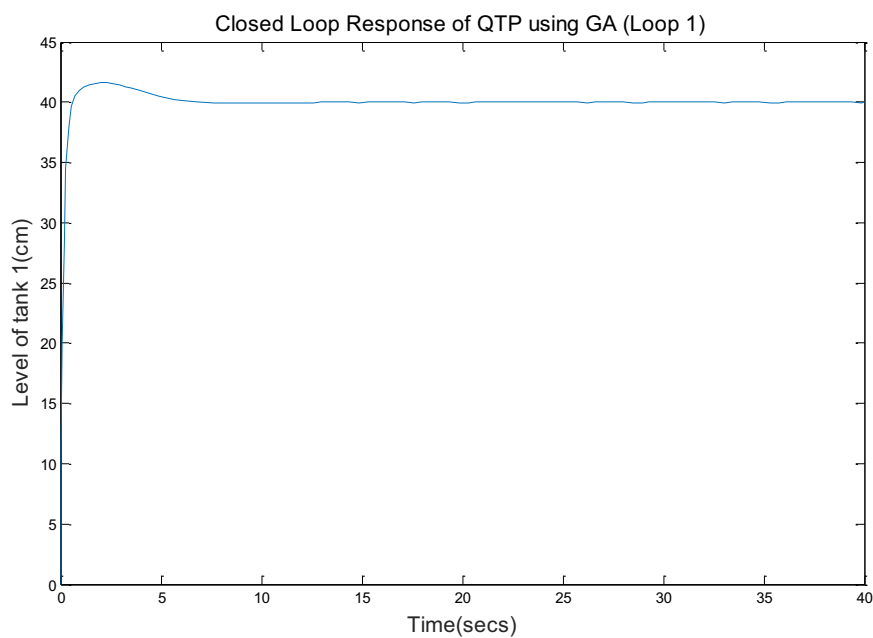


Figure 17. Closed loop Response of QTP using GA for 1st loop

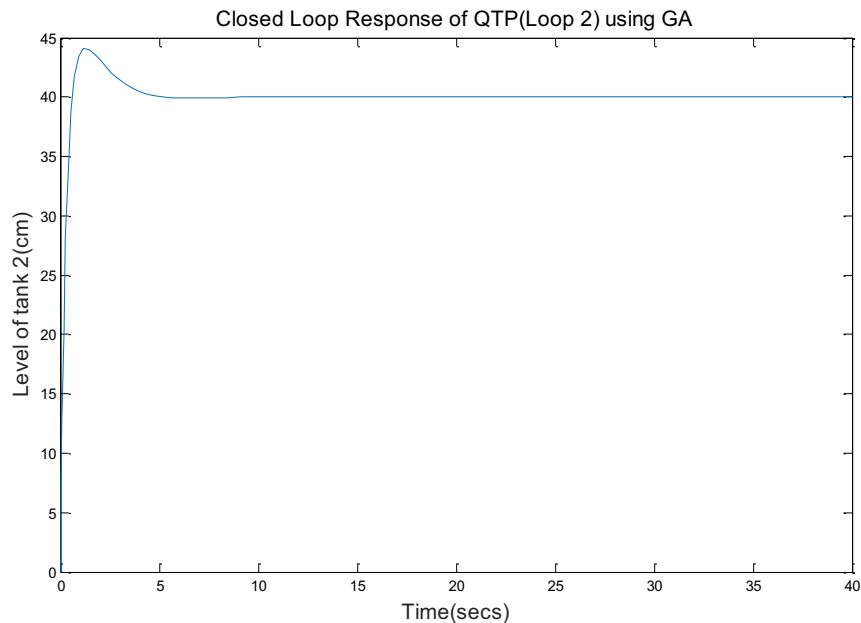


Figure 18. Closed loop Response of QTP using GA for 2nd loop

The response of CSTR model is shown in figure (7) – (12). Similarly the response of both the loops of QTP model is shown in figure (13) – (18).

5. Conclusions

In this work, we used two different optimization techniques for tuning of PID controller parameters for two process is discussed and we infer that, in CSTR the PSO based tuned values gives better response than that of GA. Whereas in QTP process both the optimization techniques gives almost similar response with slight variation in their peak Overshoot values.

References

- [1] Abubeker Yimam, Venkatanarasaiah and Tassisa Kaba 2004 Adaptive Control Design for a MIMO Chemical Reactor *Thesis report* 15 – 26
- [2] Biswajit jena and Sagar kumar 2011 Tuning of PID controller by bio inspired techniques *ethesis* p 14
- [3] Ademu Victor Okpanachi 2010 Developing Advanced Control strategies for a 4-Tank Laboratory process *Thesis report* 11 – 20
- [4] Stephanopoulos G, *Chemical Process Control: An Introduction to Theory and Practice* 1st edition (Prentice Hall PTR)
- [5] Coughanowr R. Donald 1965, *Process Systems Analysis and Control* (McGraw-Hill International Editions, Inc.) 82 - 86
- [6] Sivagamasundari S 2016 Design and Implementation of Controllers for a CSTR Process

- International Journal of Emerging Technology in Computer Science & Electronics* **23(1)** 175 – 83
- [7] Nikul Maheshwari, Neha Jain, Arun Jingar and Mahendra Suthar 2016 Design And Analysis of PID Controller For CSTR Process *International Journal of Science Engineering and Technology Research* **5(2)** 600 - 03
- [8] <http://www.jcbrolabs.org/matlab>
- [9] <https://www.scribd.com/document/111102573/Identification-and-Optimization-of-Pi-Parameters-Based-on-Genetic-Algorithm-for-Non-Linear-System-Using-Matlab>
- [10] Mohd rahairi bin rani 1986, Multi-objective optimization of PID parameters using genetic algorithm *Thesis* 15 -23