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Research paper



# Unsteady MHD Free Convection by Passing Cobalt Nanoparticles Past an Accelerated Vertical Plate Through Porous Medium of Ethylene Glycol

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### Abstract

This article studies , the effects of unsteady magneto-hydrodynamic boundary layer when cobalt nanoparticles where passed onto a vertical plate which is given an impulse by exponential acceleration through porous medium of ethylene glycol when thermal radiation is present ,parameters of absorption of heat and radiation, chemical reaction parameter, magnetic field in transverse direction are theoretically studied. We consider cobalt nanoparticles resembles to spherical structure and range of volume of nanoparticle concentration is basically less than or equal to 4%. Considering the boundary conditions we have formulated a governing equations of nanoparticles which is in the form of partial differential equations. The precise solutions for velocity, concentration and temperature profiles are obtained by plotting the graphs of the equations formed using Laplace Method using MATLAB software. This study of heat transfer in nanoparticles finds application in tribological aspect, biological fields dealing with molecular level cell interactions, improving thermal conductivity and commercial cooling applications. The most significant outcome of this study is found in the concentration profile where time is varied. We see that there is 68% decrease in concentration when the time interval chosen is 0.2, whereas the decrease is found to 55% when the interval is increased to 0.4 keeping the distance from vertical plate constant.

Keywords: Cobalt Nanoparticles; Ethylene Glycol; Laplace Transform; MHD; Porous Medium

# 1. Introduction

Several industry-related processes deal with the heat transfer by way of flowing fluid either laminar or turbulent system in addition to stagnant or flowing boiling fluids. These processes include wide-range of pressures & temperatures. Most of these applications would get benefited from the decrease in thermal resistance of heat transfer fluid. This condition would lead to decrease in heat transfer systems with lesser capital costs and improvements in energy efficiencies with the latest technology and improvements, solid particles have diameters less than 100 nm can be produced with nanotechnology. Due to this, liquid suspensions can be developed by diffusing the nanoparticles in base fluid in place of a millimeter or micro-sized particles and it can be used them for improving the heat transfer (Masuda et al [6] and Choi [4]). The liquid suspensions can also be called as Nano fluids. As nanoparticles are minuscule, they act like fluid molecules which resolves the main problem of small passages getting clogged when larger particles are used. This theoretical models has been proposed by several researchers to describe and foretell these aberrant ratios of thermal conductivity which is called as the effective thermal conductivity of Nano fluid divided by base fluid's thermal conductivity.

Considering the parameter of heat generation and absorption MHD free convection of nanoparticles passing over a plate was studied by Chamkha and Aly [2]. Using Keller box method Chamkha et al. [3] studied Nusselt and Sherwood number and radiation effects through the pervious medium consisting nanofluids on a vertical plate. While the velocity and temperature profiles for inclined plate were studied by Akilu and Narahari [1] and [7]. A rotating vertical plate system was considered by Satya Narayana et al. [8] and [9] in which they studied thermal-radiation and heating effects for MHD nanofluid flow. Turkyilmazoglu [11] and [10] found out the solution for closed form analytic problems for an accelerated plate considering radiation effects. Kumaresan et al. [5] found a precise solution for MHD convection for silver nanoparticles suspended in water as a base fluid. By studying all these cited articles we found out a solution for Magneto Hydrodynamic boundary layer flow for cobalt nanoparticles passed over a vertical plate through ethylene glycol which is porous medium. We have considered here the presence of parameters of absorption of radiation and heat generation, magnetic field, chemical reaction parameter to study the velocity temperature and concentration profile. The governing equation for flow of nanofluids over a vertical plate was solved by Laplace Transform method considering appropriate boundary condition and the graphs were plotted by varying the various parameters and their influence on the characteristic curves were discussed.

# 2. Mathematical Analysis

A nanofluid is considered to be possessing properties like viscous, incompressible and electrically conducting. It is allowed to flow over a vertical plate in exponential manner. We have applied transverse magnetic field over a plate whereas radiation absorption,



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chemical reaction parameters are also considered. A plate is placed such that x axis is vertical and Y axis is horizontal. The temperature and concentration initially is said to be  $T\infty$  and  $C\infty$ . For t >0, the plate is allowed to experience an acceleration of u =u0exp(a't') in upward direction. Magnetic force B0 is considered to be operated parallel to the ordinate system. We have assumed the plate to be in thermal equilibrium for both fluid and nanoparticles. There is no electric field present in the considered system as no applied voltage is present. The plate is placed on y=0 and flow has boundary layer at y>0. The variation in pressure gradient is neglected in the considered system. Heat flux which causes radiation is applied on plate in perpendicular direction. The flow variables that we are considering are functions of y and t. The above explained coordinate system is illustrated in fig.1 along with various boundary layers present at y>0.

The governing equations of this investigation are given by:

$$\rho_{qr} \frac{\partial u}{\partial t'} = \mu_{qr} \frac{\partial^2 u}{\partial t^2} + (\rho \beta)_{qr} g \left( T' - T_{\pi}^{*} \right) - \sigma B_0^{-2} u - \frac{v_f u}{k}$$
(1)

$$\frac{\partial I}{\partial t'} = \frac{1}{(\rho C_F)_{q'}} \left[ k_{q'} \frac{\sigma I}{\partial t'} - \frac{\partial q_F}{\partial y} - Q_E \left(T' - T_{u'}^{-}\right) - \right] + Q_E^{1} \left(C' - C_{u'}^{-}\right)$$
(2)

$$\frac{\partial C'}{\partial r'} = D \frac{\partial^2 C'}{\partial r'} - k_r \left(C' - C_{\pi}^{-1}\right)$$
(3)

The initial boundary conditions are given by

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$$r' \le 0$$
:  $u = 0, T' = T_u^-, C' = C_u^-$  for all  $y$   
 $r' > 0$ :  $u = u_0 \exp(a'r'), T' = T_u^- + (T_u^- - T_u^-)Ar', C' = C_u^- + (C_u^- - C_u^-)Ar'$  at  $y = 0$   
 $u \to 0, T' \to T_u^-, C' \to C_u^-$  as  $y \to \infty$  where grig cos(alpha) (4)  
Where  $A = \frac{u_u^{-2}}{v_u}$ . For an optically thick fluid, we can adopt Rosseland approximation for radiative flux

$$q_r$$
 by the expression  $q_r = -\frac{4\sigma'}{3k'}\frac{\partial T^{*4}}{\partial y}$ 
  
(5)

where  $\sigma'$  is the Stefan-Boltzmann constant and k' the Rosseland mean absorption coefficient. We assumed that the temperature differences within the flow is sufficiently small such that T'' may be expressed as a linear function of the temperature. This is obtained by expanding T'' in a Taylor series about a free stream temperature  $T'_{i}$  as follows:

Neglecting higher-order terms in Eq. (6) beyond the first order in  $(T' - T'_{\pi})$ , we get

$$T^{\prime 4} \simeq 4T_{a}^{\prime 2}T' - 3T_{a}^{\prime 4}$$
(7)

In view Eqs. (6) and (7) reduces to

$$\frac{\partial T'}{\partial t'} = \frac{1}{(\rho C_F)_{q'}} \left[ k_{q'} + \frac{16\sigma' T_u^{d}}{3k^*} \right] \frac{\partial^2 T'}{\partial \gamma^2} - \frac{Q_0 \left(T' - T_u'\right)}{(\rho C_F)_{q'}} + Q_1' \left(C' - C_u'\right)$$
(8)

Introducing non-dimensional variables

$$U = \frac{u}{u_0}, Y = \frac{yu_0}{v_f}, t = \frac{t'u_0^1}{v_f}, M = \frac{\sigma B_0^2 v_f}{\rho_f u_0^2}, G_T = \frac{g \beta_f v_f (T_u^- - T_u^-)}{u_0^3}, \theta = \frac{T' - T_u^-}{T_u' - T_u'}, C = \frac{C' - C_u^-}{C_u' - C_u^-}, N = \frac{16\sigma' T_u^{-6}}{3k' k_f}, \Pr = \frac{v_f (\rho C_p)_f}{\kappa_f}, k = \frac{v_f K_r}{u_e^2}, Sc = \frac{v_f}{D}, Q = \frac{v_f Q_0}{u_e^2 (\rho C_p)_f}, Q_1 = \frac{v_f Q_1' (C_u^- - C_u^-)}{u_e^2 (T_u^- - T_u^-)}$$
(9)

1.22

$$\begin{split} &-\frac{A_{1}\exp(-a_{1})}{2}\bigg[\exp[T\sqrt{a_{1}AQ}-a_{1}A]\exp\left(\frac{T\sqrt{A}}{2\sqrt{b}}+\sqrt{a_{1}Q}-a_{1})t\right]+\exp\left(-T\sqrt{a_{1}AQ}-a_{1}A\right)\exp\left(\frac{T\sqrt{A}}{2\sqrt{b}}-\sqrt{a_{1}Q}-a_{1})t\right)\bigg]\\ &-\frac{A_{1}}{2}\bigg[\exp\left(T\sqrt{kSc}\right)\exp\left(c\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(c\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}\right)\right)\bigg]\\ &+A_{2}\left[\left(\frac{t}{2}+\frac{T\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(T\sqrt{kSc}\right)\exp\left(c\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\left(\frac{t}{2}+\frac{T\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(-T\sqrt{kSc}\right)\exp\left(c\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}\right)\right)\right]\\ &-\frac{A_{1}}{2}\left[\exp\left(T\sqrt{kSc}-a_{1}Sc\right)\exp\left(c\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\left(\frac{t}{2}+\frac{T\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(-T\sqrt{kSc}\right)\exp\left(c\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}\right)\right)\right]\\ &+\frac{A_{1}}{2}\left[\exp\left(T\sqrt{kSc}-a_{1}Sc\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}-a_{1})t\right)+\exp\left(-T\sqrt{kSc}-a_{1}Sc\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}-a_{1})t\right)\right]\\ &+\frac{A_{1}}{2}\left[\exp\left(T\sqrt{kSc}-a_{1}Sc\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}-a_{1})t\right)+\exp\left(-T\sqrt{kSc}-a_{1}Sc\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}-a_{1})t\right)\right]\\ &+\frac{A_{2}}{2}\left[\exp\left(T\sqrt{kSc}-a_{2}Sc\right)\exp\left(\frac{T\sqrt{a_{2}AQ}}{2\sqrt{b}}\right)\exp\left(\frac{T\sqrt{A}}{2\sqrt{b}}+\sqrt{bt}-a_{1})t\right)+\exp\left(-T\sqrt{a_{2}AQ}\right)\exp\left(\frac{T\sqrt{A}}{2\sqrt{b}}-\sqrt{bt}-a_{1})t\right)\right]\\ &+\frac{A_{2}}{2}\left[\exp\left(T\sqrt{a_{2}AQ}\right)\exp\left(\frac{T\sqrt{a_{2}AQ}}{2\sqrt{b}}+\sqrt{a_{2}QT}\right)+\exp\left(-T\sqrt{a_{2}AQ}\right)\exp\left(\frac{T\sqrt{A}}{2\sqrt{b}}-\sqrt{a_{2}QT}\right)\right)\right]\\ &+\frac{A_{2}}{2}\left[\exp\left(T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{A}}{2\sqrt{b}}-\sqrt{bt}\right)\right]\\ &-A_{2}\left[\left(\frac{t}{2}+\frac{T\sqrt{Sc}}{4\sqrt{b}}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{A}}{2\sqrt{b}}-\sqrt{bt}\right)\right]\\ &+\frac{A_{2}}}{2}\left[\exp\left(T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{A}}{2\sqrt{b}}-\sqrt{bt}\right)\right]\\ &+\frac{A_{2}}}{2}\left[\exp\left(T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}\right)\right]\\ &+\frac{A_{2}}}{2}\left[\exp\left(T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}\right)\right]\\ &+\frac{A_{2}}}{2}\left[\exp\left(T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}\right)\right]\\ &+\frac{A_{2}}}{2}\left[\exp\left(T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}-\sqrt{bt}\right)\right]\\ &+\frac{A_{2}}}{2}\left[\exp\left(T\sqrt{kSc}\right)\exp\left(T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)\right]\\ &+\frac{A_{2}}}{2}\left[\exp\left(T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)+\exp\left(-T\sqrt{kSc}\right)\exp\left(-T\sqrt{kSc}\right)\exp\left(\frac{T\sqrt{Sc}}{2\sqrt{b}}+\sqrt{bt}\right)\right)\right]\\ &+\frac{A_{2}}}{2}\left[\exp$$

#### The properties of nanofluids are given as follows

$$\mu_{qf} = \frac{\mu_f}{(1-\varphi)^{21}}, \rho_{qf} = (1-\varphi)_{gf} + \varphi \rho_1, (\rho C_p)_{qf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_f,$$

$$(\rho \beta)_{qf} = (1-\varphi)(\rho \beta)_f + \varphi(\rho \beta)_f, \quad k_{qf} = k_f \left[\frac{k_f + 2k_f - 2\varphi(k_f - k_f)}{k_f + 2k_f + \varphi(k_f - k_f)}\right]$$
(10)
The Equations (1)-(3) are reduced into the following forms by using equations (8) and (9) as follows
$$(1-\varphi) = \frac{1}{2} \left[\frac{k_f + 2k_f - 2\varphi(k_f - k_f)}{k_f + 2k_f + \varphi(k_f - k_f)}\right]$$

$$\frac{\partial U}{\partial t} = \alpha_1 \frac{\partial^2 U}{\partial Y^2} + \alpha_2 Gr \theta - \alpha_3 \left( M + \frac{1}{K} \right)$$
(11)

$$\frac{\partial \theta}{\partial t} = a_4 \frac{\partial^2 \theta}{\partial T^2} - a_5 Q \theta + Q_1 C$$
(12)

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial T^2} - kC$$
(13)

where

$$\begin{split} x_1 &= (1-\varphi) + \varphi \left(\frac{\rho_i}{\rho_f}\right) , x_2 &= (1-\varphi) + \varphi \left(\frac{(\rho \mathcal{G})_i}{(\rho \mathcal{G})_f}\right) , x_3 &= (1-\varphi) + \varphi \left(\frac{(\rho \mathcal{C}_f)_z}{(\rho \mathcal{C}_p)_f}\right) \\ x_4 &= \frac{k_2 + 2k_f - 2\varphi(k_f - k_z)}{k_z + 2k_f + \varphi(k_f - k_z)} , \alpha_1 = \frac{1}{(1-\varphi)^{1/3}} x_i \ \alpha_2 &= \frac{x_1}{x_1} , \alpha_1 = \frac{1}{x_1} , \alpha_2 = \frac{x_4 + N}{x_5 Pr}, \alpha_3 = \frac{1}{x_1} \end{split}$$

#### In non-dimensional form, the conditions are reduced to as follows

$$t \le 0: U = 0, \quad \theta = 0, \quad C = 0 \text{ for all } Y$$
  
 $t > 0: U = \exp(at), \theta = t, \quad C = t \quad at \quad Y = 0$  (14)  
 $U \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad as \quad Y \rightarrow \infty$ 

#### Solution of the problem

The non-dimensional equations (11)-(13) associated with the boundary conditions (14) are solved by the method of Laplace transforms technique and the solutions of velocity, temperature and concentration are given below

$$\begin{split} &U = \frac{exp(at)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 + aB} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_2 P + a} t \right) + exp\left( -T \sqrt{a_3 B^2 + aB} \right) erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} - \sqrt{a_2 P + a} t \right) \right] \\ &- \frac{A}{2} \left[ exp\left[ T \sqrt{a_2 B^2} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P t} \right) + exp\left( -T \sqrt{a_3 B^2} \right) erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} - \sqrt{a_2 P t} \right) \right] \\ &+ A_1 \left[ \left[ \frac{t}{2} + \frac{TB}{4\sqrt{a_1 B^2}} \right] exp\left[ T \sqrt{a_3 B^2} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P t} \right) + \left( \frac{t}{2} - \frac{TB}{4\sqrt{a_2 B^2}} \right) erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} - \sqrt{a_2 P t} \right) \right] \\ &+ A_1 \left[ \frac{t}{2} + \frac{TB}{4\sqrt{a_1 B^2}} \right] exp\left[ T \sqrt{a_3 B^2} - a_1 B \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P - a_1} t \right) \right] \\ &+ A_1 \frac{exp(-a_1)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P - a_1} t \right) \right] \\ &+ \frac{A_1 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P - a_1} t \right) \right] \\ &+ \frac{A_1 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{B}}{2\sqrt{t}} + \sqrt{a_1 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{A}}{2\sqrt{t}} + \sqrt{a_2 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{A}}{2\sqrt{t}} + \sqrt{a_2 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{A}}{2\sqrt{t}} + \sqrt{a_2 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{A}}{2\sqrt{t}} + \sqrt{a_2 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 B^2 - a_1 B} \right] erfe\left( \frac{T \sqrt{A}}{2\sqrt{t}} + \sqrt{a_2 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 A^2 P - a_1 B} \right] erfe\left( \frac{T \sqrt{A}}{2\sqrt{t}} + \sqrt{a_2 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2)}{2} \left[ exp\left[ T \sqrt{a_2 A^2 P - a_1 B} \right] erfe\left( \frac{T \sqrt{A}}{2\sqrt{t}} + \sqrt{a_2 P - a_1} t \right) \right] \\ &+ \frac{A_2 exp(-a_2$$

Where

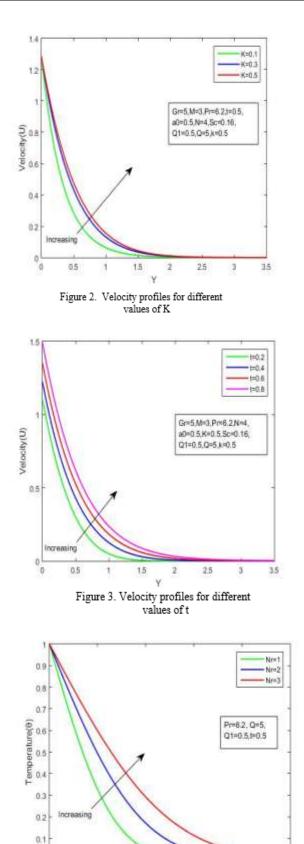
$$\begin{split} P &= M + \frac{1}{K}, \ A = \frac{1}{a_1}, B = \frac{1}{a_1}, a_1 = \frac{AQ_1}{(Sc - A)}, a_2 = \frac{kSc - Aa_5Q}{(Sc - A)}, a_2 = \frac{a_2BGr}{(A - B)}, \\ a_4 &= \frac{Aa_5Q - a_3BP}{(A - B)}, a_5 = \frac{a_1a_2BGr}{a_1^{-2}(A - B)}, a_6 = \frac{a_1a_2BGr}{a_2(A - B)}, a_7 = \frac{a_1a_2BGr}{a_2^{-1}(Sc - B)}, a_1 = \frac{kSc - a_3BP}{(Sc - B)}, \\ a_9 &= \frac{a_1a_2BGr}{a_2(Sc - B)}, A_1 = \frac{a_1^{-2}(a_3 + a_4s_3 + a_5) - a_4^{-2}(a_3a_4 + a_5)}{a_4^{-2}a_4^{-1}}, A_2 = \frac{a_1(a_3 + a_4s_3) - a_4a_5}{a_4a_5}, \\ A_3 &= \frac{a_1(a_2 - a_4) - a_3(a_1 - a_5)}{(a_2 - a_4)(a_3 - a_5)}, A_4 = \frac{a_2(a_3 + a_4s_3 + a_5) - a_4(a_3 + a_5)}{a_4^{-2}(a_2 - a_4)}, A_5 = \frac{a_3a_4a_5 + a_5(a_3 - a_5)}{a_4^{-2}(a_2 - a_4)}, \\ A_6 &= \frac{a_3 + a_4a_5 + a_5}{2}, A_6 = \frac{a_3 + a_4}{2}, A_8 = \frac{a_5}{2}, A_8 = \frac{a_3a_4 + a_5}{2}, A_8 = \frac{a_5}{2}, A_8 = \frac{a_5}{$$

Thermal and physical properties of ethylene glycol based nano particles are-

Thermo-physical proper- ties	Base fluid	Cobalt	ZnO
$\rho (kg/m^3)$	1110	8900	5606
Cp (J/kg K)	2382	419.8	514
k (W / mK)	0.258	99.2	2.5
$\beta x \ 10^5 \ (K^{-1})$	5.7	1.38	1.57
φ	0.00	0.04	0.07

## 3. Results and Discussion

We have studied the effect of cobalt nanoparticles suspended in ethylene glycol chosen as the base fluid. The parameters chosen for study were the concentration, temperature and velocity profiles. The above parameters were thoroughly discussed and the graphs were plotted to demonstrate the behavioral characteristics. The values chosen were a0=0.5,Q1=0.5,K=0.5,M=3,k=0.5,N=4,Pr=20,,Gr=5 ,Sc=0.16 and t=0.5. There parameters were varied in each of the above mentioned profiles and characteristics were determined using MATLAB. The table depicts the thermo-physical properties considering that the cobalt nanoparticles resembles to spherical structure. The nanoscale characteristics are invalidated when the  $\phi$ value approaches zero. We have also found out the variation of velocity profile with varying parameter of heat generation Q. The characteristic curves shows the steady increment in slope as Q value increases in interval of 1 to 5. This implies that near the boundary layer region the velocity tends to retard with increase in heat absorption. The variation of velocity with varying thermal radiation parameter N was also studied. The curves show that the velocity increases as the N value increases. As thermal radiation increases boundary layer near plate also increases leading to lesser cooling rates. The effect of magnetic field parameter on the velocity profile was also studied. The graph shows that the velocity decreases steadily as the M value increases. Development of Lorentz force causes slowing down of Nano fluid particles in boundary layer. The reduction in drag force causes increment in dimensionless parameter of permeability K which causes reduction in velocity profile evidently demonstrated. It is observed that there is gradual increase in velocity as the time increases when the graph is plotted. When temperature profile is studied the following factors have a significant effect on shifting the characteristic curves. It is found that there is positive correlation between the temperature profile and the corresponding varying parameter. These parameters include thermal radiation parameter (N), time constraint (t), and radiation absorption parameter (Q1) especially the correlation being very dominant when time is varied as we can see distinct increase in characteristic curve. Whereas when heat absorption parameter is concerned there is a negative effect of increase in Q thus we get curves with increasing slope. As concentration is also one of the prevalent factor in the study of nanofluids we have also studied concentration profile with variation Schmidt (Sc) number, non-dimensionless time (t) and chemical reaction parameter (k). As the Schmidt number (Sc) and chemical reaction parameter (k) is increased, the concentration profile shifts towards the origin which is evident from our study. Considering the concentration profile with variation in time the results where completely opposite of the previous scenario. It signifies that as the nanoparticles are allowed to flow near the boundary layer over the period of time, the concentration profile shifts away from origin as a result of our study.



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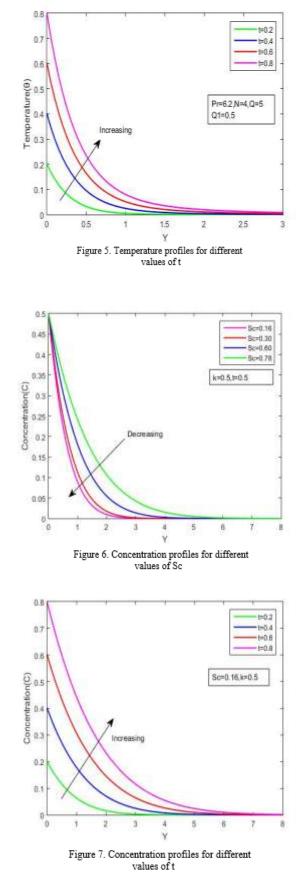
0.5

1.5

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Figure 4. Temperature profiles for different values of Nr

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## 4. Conclusions

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The motive of our research is to study the effects of magnetohydrodynamic boundary layer when cobalt nanoparticles where passed onto a vertical plate through porous medium of ethylene glycol. We have considered the parameters of absorption of heat and radiation. Laplace Transform method is used to solve the partial differential equations which governs the flow of cobalt nanoparticles near the boundary layer. The parameters under study in this paper are parameter of magnetic field, porosity parameter , Grashof number ,chemical reaction parameter, Schmidt number, parameter of volume fraction .We have arrived at following conclusion-

•As the heat absorption (Q) climbs, so do the nanofluid velocity while there a dip down in temperature profile as heat absorption increases. Thermal radiation parameter (N) goes up velocity increases; and the same characteristic is followed for the temperature profile.

•As the plot of velocity profile with variation in magnetic parameter (M) is plotted, we deduce that with a steady increase in M there is reduction in velocity. Dimensionless time (t) has a positive correlation with all the three dimensionless profiles considered in this paper.

•Chemical reaction parameter (k) is considered for concentration profile and graphs showed that there is inverse relationship between k and concentration profile. Schmidt number plays a significant role in determination of concentration profile of a nanoparticle which shows there is decrement in concentration as the Schmidt number increases.

•As there is gradual increase in porosity parameter of a nanofluid the velocity profile is found to be increasing. These results find wider applications in the field of transportation engineering and cooling in commercials sectors.

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