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Weaker form of totally continuous functions

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Abstract: The objective of this paper is to study new type of continuous functions called totally αgs-continuous functions using αgs-open sets. Furthermore we discuss covering properties and obtain their characterizations by including counter examples.

Keywords: Totally αgs-continuous, αgs-closed set, αgs-continuous, αgs-connected.

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1. Introduction

General topology plays an important role in many fields of applied sciences as well as branches of mathematics. In general topology, generalized open sets plays an important role. Indeed a significant theme in general topology and real analysis concerns that variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets.

Continuous functions stand among the most fundamental concepts in the whole mathematical science. Many different forms of stronger and weaker forms of such functions have been introduced and studied over the years in the field of general topology. The idea of totally continuous functions presented in 1980 by Jain [1]. In [2] totally semi continuous functions presented as a generalization of totally continuous functions and basic results were proved. Recently, Rajamni and Vishwanathan [3] introduced the notion of αgs-closed set using α-closure operator. Using αgs-closed set, almost contra αgs-continuous as well as contra ags-continuous functions were introduced in [4].

In this paper the new generalization of total continuity named as totally αgs-continuous is presented. The notion of totally αgs-continuous is a weaker form of total continuity. Also it's fundamental properties are investigated.

2. Preliminaries

In the entire paper (X, τ), (Y, σ)(or simply X, Y) represents topological spaces on which no separation axioms are assumed unless explicitly stated. $Cl(A)$ and $Int(A)$ represents closure and interior of A with respect to τ in the sequel. We recall some definitions which are necessary.

Definition 1. A subset A of a topological space X is known as:

1. semi open set [5] if $A \subset Cl(Int(A))$.
2. semi closed set [6] if $Int(Cl(Int(A))) \subset A$.
3. α-open [7] if $A \subset Int(Cl(Int(A)))$.

Definition 2. [3] A subset A of X is αgeneralized semi-closed (shortly, αgs-closed) set if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is semi open in X.
The complement of αgs-closed set is αgeneralized-semi open (shortly,αgs-open). The family of all αgs-closed sets of X is denoted as αGSC(X,τ) besides αgs-open sets by αGSO(X,τ).

Definition 3. [8] The intersection of all αgs-closed sets containing a set A is said to be αgs-closure of A and is denoted by αgs$Cl(A)$. A set A is αgs-closed if and only if αgs$Cl(A) = A$.

Definition 4. [8] The union of all αgs-open sets contained in A is said to be αgs-interior of A and is denoted by $\alpha gsInt(A)$. A set A is αgs-open if and only if $\alpha gsInt(A) = A$.

Definition 5. [9] A topological space X is said to be αgs-T_2 if for each pair of distinct points x and y of X, there exists disjoint αgs-open sets, one containing x and the other containing y .

Definition 6. [1] A function $h : X \to Y$ is said to be totally continuous if the inverse image of each open subset of Y is a clopen subset of X.

Definition 7. [4] A topological space X is said to be αgs-connected if it cannot be written as the union of two non-empty disjoint αgs-open sets.

3. Totally αgs-Continuous Functions

In this section, the idea of a new class of functions named totally αgs-continuous function is presented. In addition, the relationships with existing functions are discussed.

Definition 8. A function $\mu : X \to Y$ is said to be totally αgs-continuous if $\mu^{-1}(K)$ is αgs-clopen in X for every open set K of Y.

Remark 1. Every totally continuous function is totally αgs-continuous. The converse may not be true.

Example 1. Consider topologies $\tau = \{X, \phi, \{p\}, \{q,r\}\}$, $\sigma = \{Y, \phi, \{p\}, \{p,q\}\}$ on $X = Y = \{p,q,r\}$, We have αGSC(X)= $\{\{q\}, \{r\}, \{p,q\}, \{p,r\}\}$. Define a function $h : X \to Y$ by $h(p) = q, h(q) = p, h(r) = r$. Then h is totally αgs-continuous but it is not totally continuous as $h^{-1}(\{p\}) = \{q\}$ is not clopen in X.

Remark 2. Every totally αgs-continuous is αgs-continuous function. But the converse is not true as illustrated by the following example.

Example 2. Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{a\}, \{a,b\}\}$. We have αGSC(X) are $\{\{c\}, \{b,c\}\}$. Let $Y = \{1,2,3\}$, $\sigma = \{Y, \phi, \{1,2\}\}$. Define a function $h : X \to Y$ by h(a) = 2, f(b) = 1, f(c) = 3. Then h is totally αgs-continuous but it is not αgs-continuous as $h^{-1}(\{1,2\}) = \{a,b\}$ is αgs-open set but not αgs-closed set in X.

Theorem 9. *If f is totally αgs-continuous function from αgs-connected space X onto any space Y, then Y is an indiscrete space.*

Proof. Assume that Y isn't indiscrete. Let K be a proper non-empty subset of Y. Then $f^{-1}(K)$ is non-empty αgs-clopen subset of X, which is contradiction to the fact that X is αgs-connected space. Then Y is indiscrete space. □

Theorem 10. *Let X be αgs-connected with Y be T_1. If $h : X \to Y$ is totally αgs-continuous, then f is constant.*

Proof. Let X be a αgs-connected and h as totally αgs-continuous. Since Y is T_1-space, $\Lambda = \{h^{-1}(y) : y \in Y\}$ is a disjoint αgs-clopen partition of X. If $|\Lambda| \geq 2$, then there exists a proper αgs-clopen set M for some $U \in \Lambda$ in αgs-connected space X. This is contradiction to the fact that X is αgs-connected. So $|\Lambda| = 1$. Therefore f is constant. □

Definition 11. Consider a topological space X. We define an equivalence relation on X by taking $p \approx q$ if there is a αgs-connected subset of X containing p as well as q. The equivalence families are said to be αgs-separation of X or αgs-component of X.

Theorem 12. *Let $h : X \to Y$ be a totally αgs-continuous function from a topological space X into T_1 space Y. Then h is constant on each αgs-component of X.*

Proof. This result follows immediately from above theorem. □

Definition 13. For a subset A of X, αgs$Cl(A) - \theta$gs$Int(A)$ is said to be αgs-frontier of A and is denoted as αgs-$Fr(A)$.

Theorem 14. *The set of all points p of X at which $h : X \to Y$ is not totally αgs-continuous is identical with the union of αgs-frontier if the inverse images of closed sets of Y containing $h(x)$.*

Proof. Assume that h is not totally αgs-continuous at $p \in X$, then there exists an open set V of Y containing $h(x)$ such that $h(U)$ is not contained in V for each αgs-clopen set U containing p. This indicates $U \cap (X - h^{-1}(V)) \neq \phi$ for each $U \in \alpha GSO(X, p)$. Therefore, $p \in \alpha gsCl(X - h^{-1}(V))$. But, as $p \in h^{-1}(V) \subset$ αgs$Cl(h^{-1}(V))$, $p \in \alpha$gs$Cl(h^{-1}(V)) \cap \alpha$gs$Cl(X - h^{-1}(V))$. This shows that, $p \in \alpha gs\text{-}Fr(h^{-1}(V))$.
Conversely, assume $p \in \alpha gs\text{-}Fr(h^{-1}(V))$ for some open set V of Y containing $h(x)$ with h is totally αgs-continuous at $p \in X$. Then there exists $U \in \alpha GSO(X, p)$ such that $h(U) \subset V$. This indicates $x \in U \subset h^{-1}(V)$. Therefore $p \in \alpha gsInt(h^{-1}(V)) \subset X - \alpha gs\text{-}Fr(h^{-1}(F))$. This is contradiction to the fact that, $x \in \alpha gs\text{-}Fr(h^{-1}(F))$. Therefore h is not totally αgs-continuous. □

Definition 15. A filter base Ω in a topological space X is said to be αgs-co-convergent to a point q in X if for any $U \in \alpha GSO(X)$ containing q, there exists $K \in \Omega$ such that $K \subset U$.

Theorem 16. *If a function $h : X \to Y$ is totally αgs-continuous, then for each point $q \in X$ and every filter base Ω in X αgs-co-converging to q, therefore filter base $h(\Omega)$ is convergent to $h(q)$.*

Proof. Let $q \in X$ as well as Ω be any filter base in αgs-co-converging to q. Since h is totally αgs-continuous, then for any open set V of Y containing $h(q)$, there exists $U \in \alpha GSO(X)$ containing q such that $h(U) \subset V$. As Ω is αgs-co-converging to q, there exists $K \in \Omega$ such that $K \subset U$. This means that $h(K) \subset V$ and therefore the filter base $h(\Omega)$ is convergent to $h(q)$. □

4. Covering Properties

Definition 17. A topological space X is said to be:

1. αgs-co-compact if every αgs-clopen cover of X has finite subcover.
2. αgs-co-compact relative to X if every cover of A by αgs-clpoen sets of X has finite subcover.
3. αgs-compact [4] if every αgs-open cover of X has finite subcover.
4. αgs-compact [4] if A is a αgs-compact as a subspace of X.

Theorem 18. *If a function $\mu : X \to Y$ is totally αgs-continuous as well as Q is αgs-co-compact relative to X, then $\mu(Q)$ is compact in Y.*

Proof. Let $\{H_i : i \in I\}$ be any cover of $\mu(Q)$ by open sets of Y. For each $x \in Q$, there exists $i_x \in I$ such that $\mu(x) \in (H_\alpha)_x$ and there exists $U_x \in \alpha GSO(X)$ containing x such that $\mu(U_x) \subset (H_\alpha)_x$. Since the collection $\{U_x : x \in Q\}$ is a cover of Q by αgs-clopen sets of X, there exists a finite subset Q_0 of Q such that $Q \subset \bigcup\{U_x : x \in Q_0\}$. So, we get $\mu(Q) \subset \bigcup\{\mu(U_x) : x \in Q_0\} \subset \bigcap\{(H_\alpha)_x : x \in Q_0\}$ and hence $\mu(Q)$ is compact. □

Theorem 19. *Following results are equivalent for a function $h : X \to Y$*

1. *h is totally αgs-continuous,*
2. *for every open set V of Y, $h^{-1}(V)$ is αgs-clopen in X,*
3. *for every closed set F of Y, $h^{-1}(F)$ is αgs-clopen in X.*

Proof. **1⇒2** Consider an open set V in Y as well as $x \in h^{-1}(V)$. Then $h(x) \in V$. By (1), there exists a αgs-clopen set U_x in X such that $x \in U_x \subset h^{-1}(V)$. This implies $h^{-1}(V)$ is αgs-clopen nhd of x. As x is arbitrary, $h^{-1}(V)$ is αgs-clopen nhd of each of its points. This indicates $h^{-1}(V)$ is αgs-clopen in X.
2⇒1 Let H be any open set in Y containing $h(x)$, such that $x \in h^{-1}(H)$. Again by (2), $h^{-1}(H)$ is αgs-clopen in X. Set $G = f^{-1}(H)$, then G is αgs-clopen set in X containing x with $h(G) = h(h^{-1}(H)) \subset H$. This implies h is totally αgs-continuous at $x \in X$. As x is arbitrary, it follows that h is totally αgs-continuous at each point x of X. Then, h is totally αgs-continuous.
2⇒3 Let F be a closed set in Y. Then $Y - F$ is an open set in Y. By (2), $h^{-1}(Y - F) = X - h^{-1}(F)$ is αgs-clopen

in X. This means $h^{-1}(F)$ is αgs-clopen in X.
3⇒2 Consider O be an open set in Y, then $Y - O$ is a closed set in Y. By (iii), $h^{-1}(Y - O) = X - h^{-1}(O)$ is αgs-clopen in X. This implies $h^{-1}(O)$ is αgs-clopen in X. □

Theorem 20. *If $f : X \to Y$ is totally αgs-continuous injection and Y is T_0 then X is αgs-T_2.*

Proof. Let p and q be any two distinct points in X. As f is injective, $f(p)$ as well as $f(q)$ are distinct points in Y. As Y is T_0, there exist an open set U containing say $f(p)$ but not $f(q)$, which implies, $p \in f^{-1}(U)$ and $q \notin f^{-1}(U)$. As f is totally αgs-continuous, $f^{-1}(U)$ is a αgs-clopen subset of X. Thus for two distinct points p and q of X, there exist two disjoint αgs-clopen subsets of X such that $p \in f^{-1}(U)$ and $q \notin X - f^{-1}(U)$. Therefore X is αgs-T_2. □

Theorem 21. *Let $f : X \to Y$ be totally αgs-continuous injection function with Y is T_1-space. If A is a αgs-connected subset of X, then $f(A)$ is a single point.*

Proof. Let A be a αgs-connected member of X such that $f(A)$ is not a single point. Let p and q be distinct points of subset A in X. As f is injective $f(p)$ and $f(q)$ are distinct points of $f(A)$ in Y. As Y is T_1-space, there exist open sets U and V such that $f(p) \in U$, $f(q) \notin U$ and $f(q) \in V$, $f(p) \notin V$. This indicates $p \in f^{-1}(U)$, $q \notin f^{-1}(U)$ and $q \in f^{-1}(V)$, $p \notin f^{-1}(V)$. For the reason f is totally αgs-continuous, implies $f^{-1}(U)$ and $f^{-1}(V)$ are proper αgs-clopen sets in A. This is contradiction to the fact that, A is αgs-connected subset of X. Thus, $f(A)$ is a single point. □

Definition 22. A collection $\{A_i : i \in I\}$ of αgs-clopen sets in a topological space X is said to be αgs-clopen cover of a subset A in X if $A \subset \bigcup_{i \in I} A_i$.

Definition 23. A topological space X is said to be:

1. mildly αgs-compact if every cover of X by αgs-clopen sets has a finite subcover.
2. mildly countably αgs-compact if every countable cover of X by αgs-clopen sets has a finite subcover.
3. mildly αgs-Lindelof if every cover of X by αgs-clopen sets has a countable subcover.
4. strongly S-closed [10] if every closed cover of X has a finite subcover.

Theorem 24. *If $f : X \to Y$ is totally αgs-continuous surjection with X is mildly αgs-compact, then Y is strongly S-closed.*

Proof. Let $\{V_\alpha : \alpha \in I\}$ be any closed cover of Y. As f is totally αgs-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is αgs-clopen cover of X. As X is mildly αgs-compact, there exists a finite subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. This indicates, $Y = \cup \{V_\alpha : \alpha \in I_0\}$, which is finite subcover of Y. As a result Y is strongly S-closed. □

Theorem 25. *Let $h : X \to Y$ be a totally αgs-continuous surjection. Then, the following results hold*

1. *If X is mildly αgs-compact, then Y is compact.*
2. *If X is mildly countably αgs-compact, therefore Y is countably compact.*
3. *If X is mildly αgs-Lindelöf, then Y is Lindelöf.*

Proof. (1) Let $\{V_\alpha : \alpha \in I\}$ be any open cover of Y. As h is totally αgs-continuous, $\{h^{-1}(V_\alpha) : \alpha \in I\}$ is αgs-clopen cover of X. As X is mildly αgs-compact, there exists a finite member I_0 of I such that $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. This indicates, $Y = \cup \{V_\alpha : \alpha \in I_0\}$, which is finite subcover of Y. Then Y is compact.
(2) Let $\{V_\alpha : \alpha \in I\}$ be be any countable open cover of Y. As h is totally αgs-continuous, $\{h^{-1}(V_\alpha) : \alpha \in I\}$ is countable αgs-clopen cover of X. As X is Mildly countably αgs-compact, there exists a finite subset I_0 of I such that $X = \cup \{h^{-1}(V_\alpha) : \alpha \in I_0\}$. This indicates, $Y = \cup \{V_\alpha : \alpha \in I_0\}$, which is finite subcover of Y. So Y is countably compact.
(3) Lat $\{V_\alpha : \alpha \in I\}$ be any open cover of Y. As h is totally αgs-continuous, $\{h^{-1}(V_\alpha) : \alpha \in I\}$ is αgs-clopen cover of X. As X is Mildly αgs-Lindelöf, there exists a finite subset I_0 of I such that $X = \cup \{h^{-1}(V_\alpha) : \alpha \in I_0\}$. This implies, $Y = \cup \{V_\alpha : \alpha \in I_0\}$, which is finite subcover of Y. Thus Y is Lindelöf. □

Definition 26. A topological space X is said to be

1. αgs-co-T_1 if for each pair of distinct points x and y of X there exist αgs-clopen sets G and H such that $x \in G, y \notin G$ and $y \in H, x \notin H$.
2. αgs-co-T_2 if for each pair of distinct points x and y of X, there exist disjoint αgs-clopen sets G and H such that $x \in G$ and $y \in H$.

Theorem 27. *A topological space X is αgs-co-T_1 if and only if every singleton subset $\{x\}$ of X is αgs-clopen set.*

Proof. Let X be a αgs-co-T_1 space and $p \in X$. Let $q \in X - \{x\}$. Then for $p \neq q$, there exists αgs-clopen set U_q such that $q \in U_q$ and $p \notin U_q$. Then, $q \in U_y \subset X - \{x\}$. That is $X - \{x\}$=$\cup \{U_y : y \in X - \{x\}\}$, which is αgs-clopen set. Hence $\{x\}$ is αgs-clopen set.

Conversely, assume $\{x\}$ is αgs-clopen set for every $x \in X$. Let x and $y\in X$ with $x \neq y$. Now $x \neq y$ means $y \in X - \{x\}$. Therefore $X - \{x\}$ is αgs-clopen set containing y yet not x. Similarly, $X - \{y\}$ is αgs-clopen set containing x but not y. Then X is αgs-co-T_1 space. □

Theorem 28. *If $h : X \to Y$ is totally αgs-continuous injection and Y is T_1 then X is αgs-co-T_1 space.*

Proof. Let $f : X \to Y$ is totally αgs-continuous injection with Y as T_1. For any two distinct points x_1, x_2 of X there exists distinct points y_1, y_2 of Y such that $y_1 = h(x_1)$ with $y_2 = h(x_2)$. As Y is T_1-space there exist an open sets U and V in Y such that $y_1 \in U, y_2 \notin U$ as well as $y_1 \notin V, y_2 \in V$. That is $x_1 \in h^{-1}(U), x_1 \notin h^{-1}(V)$ and $x_2 \in h^{-1}(V), x_2 \notin h^{-1}(U)$. As h is totally αgs-continuous $h^{-1}(U), h^{-1}(V)$ are αgs-clopen sets in X. Thus, for two distinct points x_1, x_2 of X there exist αgs-clopen sets $h^{-1}(U)$ and $h^{-1}(V)$ such that $x_1 \in h^{-1}(U), x_1 \notin h^{-1}(V)$ and $x_2 \in h^{-1}(V), x_2 \notin h^{-1}(U)$. Therefore X is αgs-co-T_1 space. □

Theorem 29. *If $\mu : X \to Y$ is totally αgs-continuous injection with Y is T_2 then X is αgs-co-T_2 space.*

Proof. Let $\mu : X \to Y$ is totally αgs-continuous injection with Y is T_2. For any two distinct points x_1, x_2 of X there exist distinct points y_1, y_2 of Y such that $y_1 = \mu(x_1)$ and $y_2 = \mu(x_2)$. As Y is T_2 space there exist disjoint open sets U and V in Y such that $y_1 \in U$ and $y_2 \in V$. That is $x_1 \in \mu^{-1}(U)$ and $x_2 \in \mu^{-1}(V)$. As f is totally αgs-continuous $\mu^{-1}(U)$, $\mu^{-1}(V)$ are αgs-clopen sets in X. Furthermore μ is injective, $\mu^{-1}(U) \cap \mu^{-1}(V) = \mu^{-1}(U \cap V) = \mu^{-1}(\phi) = \phi$. Thus, for two disjoint points x_1, x_2 of X there exist distinct αgs-clopen sets $\mu^{-1}(U)$ and $\mu^{-1}(V)$ such that $x_1 \in \mu^{-1}(U)$ and $x_2 \in \mu^{-1}(V)$. Therefore X is αgs-co-T_2 space. □

Definition 30. A function $h : X \to Y$ is said to be:

1. totally αgs-irresolute if preimage of a αgs-clopen member of Y is a αgs-clopen subset of X,
2. totally pre-αgs-clopen if image of each αgs-clopen subset of X is αgs-clopen.

Theorem 31. *Let $h : X \to Y$ be surjective totally αgs-irresolute and totally pre-αgs-clopen with $k : Y \to Z$ be any function. Then $k \circ h : X \to Z$ is totally αgs-continuous if and only if k is totally αgs-continuous.*

Proof. The 'if' part is trivial. To claim 'only if' part, let $k \circ h : X \to Z$ be totally αgs-continuous furthermore, let V be an open subset of Z. Then $(k \circ h)^{-1}$(V) is a αgs-clopen member of X, $h^{-1}(k^{-1}(V))$ is αgs-clopen. As h is totally pre-αgs-clopen, $h(h^{-1}(k^{-1}(V)))$ is a αgs-clopen subset of Y. So, $k^{-1}(V)$ is αgs-clopen in Y. Thus k is totally αgs-continuous. □

Definition 32. A topological space X is said to be:

1. αgs-co-regular if for each closed set F with each point $x \notin F$, there exist disjoint αgs-clopen sets U and V such that $x \in U$ and $F \subset V$,
2. strongly αgs-co-regular if for every αgs-clopen set F with a point $x \notin F$, there exist disjoint open sets U and V such that $x \in U$ and $F \subset V$,
3. αgs-co-normal if for every pair of disjoint closed sets A and B, there exists a pair of disjoint αgs-clopen sets U and V in X such that $A \subset U$ and $B \subset V$,
4. strongly αgs-co-normal if for each pair of disjoint αgs-clopen sets A with B, there exist a pair of disjoint open sets U and V in X such that $A \subset U$ and $B \subset V$.

Theorem 33. *If $h : X \to Y$ is totally αgs-continuous, closed, injection and Y is regular, then X is αgs-co-regular.*

Proof. Let F be a closed set in X and $x \notin F$. As h is closed injection $h(F)$ is closed set in Y such that $h(x) \notin h(F)$. Now Y is regular, there exist disjoint open sets G besides H such that $h(x) \in G$ as well as $h(F) \subset H$. This indicates $x \in h^{-1}(G)$ with $F \subset h^{-1}(H)$. As h is totally αgs-continuous, $h^{-1}(G)$ besides $h^{-1}(H)$ are αgs-clopen sets in X. Further $h^{-1}(G) \cap h^{-1}(H) = \phi$. Then X is αgs -co-regular. □

Theorem 34. *If $\eta : X \to Y$ is totally αgs-continuous, open, injection with X is strongly αgs-co-regular, then Y is regular.*

Proof. Let P be a closed set in Y with $y \notin P$. Take $y = \eta(x)$ being any $x \in X$, then $x \notin \eta^{-1}(P)$. As η is totally θgs-continuous, $\eta^{-1}(P)$ is αgs-clopen set in X, not containing x. As X is strongly αgs-co-regular, there exist disjoint open sets U and V such that $x \in U$ and $\eta^{-1}(P) \subset V$. That is $\eta(x) \in \eta(U)$ and $P \subset \eta(V)$. As η is open injective, $\eta(U)$ with $\eta(V)$ are disjoint open sets in Y. Therefore, Y is regular. □

Theorem 35. *If Y is normal and $k : X \to Y$ is totally αgs-continuous, closed, injection, then X is αgs-co-normal.*

Proof. Let E and F be disjoint closed sets in Y. As k closed injection $k(E)$ as well as $k(F)$ are disjoint closed sets in Y. Now Y is normal, there exist disjoint open sets G and H in order that $k(E) \subset G$ with $k(F) \subset H$. This implies $E \subset k^{-1}(G)$ with $F \subset k^{-1}(H)$. As k is totally αgs-continuous $k^{-1}(G)$ and $k^{-1}(H)$ are αgs-clopen sets in X. Further $k^{-1}(G) \cap k^{-1}(H) = \phi$. Hence X is αgs-co-normal. □

Theorem 36. *If $k : X \to Y$ is totally αgs-continuous, open, injection and X is strongly αgs-co-normal, then Y is normal.*

Proof. Let us consider the disjoint closed sets E and F in Y.As k is totally αgs-continuous, $k^{-1}(E)$ and $k^{-1}(F)$ are αgs-clopen sets in X. As X is strongly αgs-co-normal, there exist disjoint open sets U and V such that $k^{-1}(E) \subset U$ and $k^{-1}(F) \subset V$. That is $E \subset k(U)$ and $F \subset f(V)$. As k is open injective, $k(U)$ as well as $k(V)$ are disjoint open sets in Y. Therefore, Y is normal. □

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