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Wind profile detection of atmospheric radar signals using wavelets and harmonic decomposition techniques

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Abstract

A simple and successful design of algorithms using parametric and non-parametric techniques for spectral cleaning and estimation of the atmospheric radar signal is presented. With these algorithms, signal-to-noise ratio (SNR) of the radar returns could be improved and the Doppler frequency could be predicted even at greater heights and under severe weather conditions. The predicted results are compared with the conventional techniques and good improvement is reported. Copyright © 2004 Royal Meteorological Society

Keywords: wavelet denoising; STFT; atmospheric wavelets; harmonic decomposition

1. Introduction

Wind profile detection of a mesophere–stratosphere–troposphere (MST) radar signal meant the measurement of Dopplers of the signal due to scattering of the atmospheric elements. Atmospheric radar signal refers to the signal received by the radar due to the back-scattering property of the atmospheric layers — stratified or turbulent. The back-scattered signal from the atmospheric layers is very small in terms of power with which it was emitted. The received back-scattered signals, otherwise called as *radar returns*, are associated with Gaussian noise. The noise dominates the signal as the distance between the radar and the target increases and this leads to a decrease in signal-to-noise ratio. This makes the detection of the signal in the prescribed noise difficult. The detection of the signal Doppler in such dominated noise forms the primary part of the article. After the detection of signal, the signal has to be estimated. The estimation of the signal frequency and the signal power forms the second part of the article.

The radar returns are first analyzed for its frequency characteristics. This can be done by the power spectral estimation of the back-scattered signals. Power spectral estimation (Proakis and Manolakis) of a signal specifies the spectral characteristics of a signal in frequency domain. Power Spectral Density (PSD) is the Fourier transform of the autocorrelation function of the signal. The autocorrelation of the signal specifies the signal characteristics of the signal in time domain. The power spectrum estimation (Proakis and Manolakis) is proportional to the length of the data record. The longer the data, the better will be the quality of the power spectrum estimation.

The PSD estimation techniques are broadly classified into two, namely, (a) non-parametric methods, and (b) parametric methods.

1.1. Non-parametric methods

These methods make no assumption about how the data were generated and hence are called *non-parametric*. The estimates are entirely based on a finite record of data, the frequency resolution of these methods is, at best, equal to spectral width of the rectangular window of length N, which is approximately equal to $1/N$ at the -3 dB points. Here N is equal to the length of the data record. All these methods decrease the frequency resolution in order to reduce the variance in the spectral estimate. Although the spectral estimates are expressed as a function of the continuous frequency, in practice the estimates are computed at discrete frequencies on the basis of fast Fourier transform (FFT) computations.

1.1.1. Limitations of the non-parametric methods

The non-parametric power spectral estimation methods are relatively simple, well understood and easy to compute via the FFT algorithm. The following are some of the limitations of the non-parametric methods.

- These methods require the availability of long data records in order to yield the necessary frequency resolution.
- These methods suffer from the spectral leakage effects due to windowing that are inherent in the finite-length data records. The spectral leakage masks weak signals that are present in the data.
- The inherent assumption is that the autocorrelation estimate is zero for large lags that are greater than the length of the record, which is not realistic. This assumption severely limits the frequency resolution and the quality of the power spectrum estimate.

1.2. Parametric methods

In order to overcome the limitations of the non-parametric methods, the parametric methods are used. The parametric methods do not require making any assumptions as in the case of the non-parametric methods. These methods use the extrapolation technique for autocorrelations for lags greater than the length of the record. The extrapolation is possible if we have some prior information on how the data were generated. A model for the signal generation may be constructed with a number of parameters that can be estimated from the observed data. From the model and the estimated parameters, the power density spectrum implied by the model can be computed.

The modeling approach eliminates the need for window functions and the assumption that the autocorrelation sequence is zero for lags greater than the length of the record. The parametric methods provide better frequency resolution than the FFT-based, non-parametric methods. This is especially true in applications where small data records are available due to time-variant and transient phenomena.

2. Wind profile detection using non-parametric methods

2.1. Fast Fourier transform

Wind profile detection based on fast Fourier transforms relies on computing the FFT (Proakis and Manolakis) of the autocorrelation function of the data using the general purpose FFT algorithm. The FFT of the autocorrelation function specifies the spectral characteristics of the signal in frequency domain. Presently, in many of the MST radars around the world, the wind information is obtained from the power spectrum obtained using FFT. An example of the power spectrum obtained using the FFT is shown in Figure 1.

The following are some of the observations that can be inferred from the figure. The Dopplers can be easily detected at lower range bins or lower altitudes as the signal dominates the noise at these altitudes. This is due to the nearness of the target from the radar. As the radar returns come from farther atmospheric layers, due to the amount of distance they traveled back to the radar, the intervening atmospheric disturbance and very less back-scattered signal power, the SNR tends to decrease gradually to a very low value. The effect can be seen in the power spectrum at higher altitudes. This is where the detection of signal in the dominated noise becomes difficult.

2.1.1. Limitations of FFT algorithm

The following are some of the limitations of the FFT-based wind profile detection.

- Radio Interference cannot be removed.
- SNR is very low at higher altitudes causing problems in detecting the profile of the wind.
- The information contained in the wind profile is lost due to the low SNR at higher altitudes of the radar data. This information is essential as it is useful

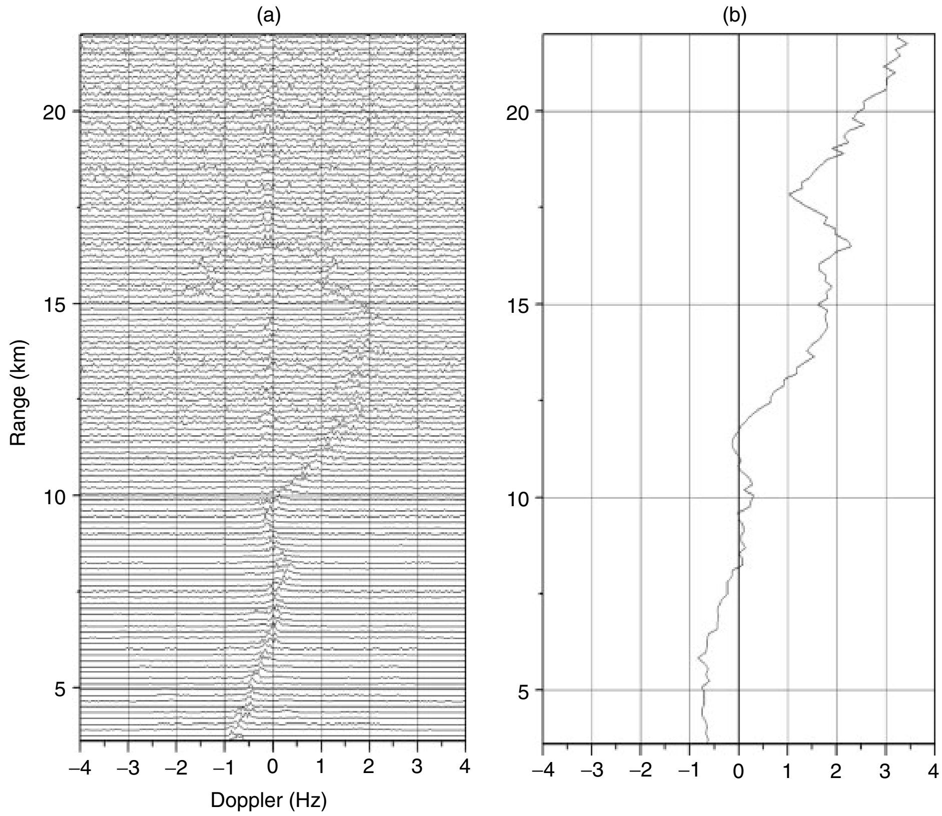

Figure 1. (a) Power spectrum plot taken on 20 July 2004 using east beam (b) Doppler profile extracted without denoising

in measuring the Doppler frequency, wind speed, radial velocities etc.

The FFT is limited in its application because of the non-usage of any spectral cleaning other than incoherent averaging at the point of reception. This averaging is not able to remove sufficient amount of noise. So a spectral cleaning technique is needed to increase the probability of detection of signal in the low SNR regions.

2.2. Wavelet denoising

From the above section, it is clear that a better spectral cleaning technique is required for improving the probability of detection of Doppler in the low SNR regions. Denoising using wavelets is one such strategy, which can be used to improve the SNR of the noise-dominated regions of the radar returns. Wavelets around the world have taken center stage in a number of applications like data compression, image processing etc., The wavelets are due for its application for denoising in atmospheric radar signal processing. An algorithm based on the wavelets for denoising the noise-dominated radar returns has been designed. The algorithm is designed to individually decide for each and every bin of the radar returns whether denoising is required by first computing the SNR of the bin. A threshold is set for the SNR below which only the denoising is performed.

Firstly, the algorithm is trained with a simulated signal (sine, cosine etc.) corrupted with different noises (Gaussian noise, random noise etc.) of different SNRs. The reason why the sine and cosine has been selected is that they are the eternal signals. A system, which works for these signals, will achieve the same for any other signal. The algorithm comes up with an optimum combination of wavelet, threshold function and level of wavelet in each of these cases. For finding the optimum combination, mean square error is calculated with the original simulated signal.

After training the algorithm in the above manner, the algorithm, with its library of optimum combination for different types of noises and different SNRs, becomes adaptive in the following sense: as the most of the noise in the radar returns is Gaussian since the signal arises mainly from the turbulent scatter, the SNR for each bin is calculated and the combination for Gaussian noise and for respective SNR is returned from the library.

Wavelets perform denoising by effectively thresholding the unwanted frequency components from the original radar returns. The thresholding of the wavelet coefficient has near-optimal noise reduction for many classes of signals. Wavelets use two different types of thresholding for denoising — soft thresholding and hard thresholding. Hard thresholding is the usual process of setting to zero the elements whose absolute values are less than the threshold. Soft thresholding is an extension of hard thresholding, where the elements whose absolute values are lower than threshold are set to zero and then the values of the remaining elements are shrunk towards zero. According to Donoho and Johnstone, soft thresholding can be used effectively in denoising schemes, provided the noise distribution is Gaussian. A common approach for threshold selection is to compute the sample variance σ^2 of the coefficients in a band and set the threshold to some multiple of the standard deviation σ. The threshold in our algorithm is set by some predefined rules, which are given later in the paper. These predefined rules use soft thresholding.

Another method of denoising is damping of unwanted frequency components. This can be done by first decomposing the noisy signal using wavelets, and damping the coefficients of the lower-level high-pass filters to zero (Cohen and Daubechies, 1990). The decomposed signal is then reconstructed with the changed coefficients. Proper care should be taken not to damp out the signal by using very high levels of decomposition. The optimum level of decomposition is obtained after experimenting several times.

The two methods mainly differ in the ways in which the noise causing high-frequency components is removed. In the first method of thresholding, the wavelet transform is calculated for the signal and the threshold is applied where the coefficients whose value falls below the threshold are made zero and the resultant signal is inverse wavelet transformed to obtain the denoised signal. The same denoising can also be done in a different way whereby the signal is decomposed using the wavelet function to different levels and damping the coefficients of the lower-level high-pass filter outputs to zero and reconstructing the signal using the changed coefficients. The reconstructed signal contains mainly the lower-frequency components, which correspond to the signal frequency.

Denoising differs from filtering schemes in the following aspects.

- Denoising does non-linear filtering i.e., filtering is made a function of some threshold function.
- Denoising tends to optimize the mean square error i.e.,

$$\frac{1}{N}\sum E[f(x_n) - f(\ldots)]^2$$

Wavelets provide a whole lot of advantages over FFT. Fourier analysis has a serious drawback. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell when a particular event took place. Wavelet analysis is capable of revealing aspects of data that other signal analysis techniques miss, aspects like trends, breakdown points, discontinuities in higher derivatives and self-similarity. Furthermore, because it affords a different view of data than those presented by traditional techniques, wavelet analysis can often compress or denoise a signal without appreciable degradation.

2.2.1. Denoising structure in wavelets

The wavelet transform provides the time-frequency representation of the signal. (There are other transforms, which give this information too, such as short time Fourier transform (STFT), Wigner distribution (Donoho, 1994) etc.). This property is used for denoising (Donoho, 1995), which can be achieved in four steps:

Step 1: Selection of wavelet combination
The SNR of the radar return is first computed and the optimum combination containing the wavelet name, level M. Thresholding rule is feedback from the library, which was created using the procedure discussed in the previous section.

Step 2: Obtaining wavelet transform coefficient
Compute the wavelet decomposition of the signal using the combination feedback from the library.

Step 3: Thresholding
The decomposed coefficients are then set to threshold using the threshold rule given by the combination. The different threshold selection rules and their specifications are given below. Each of these techniques has a unique behavior suitable for denoising a particular type or class of signals.

2.2.2. Threshold selection rules

According to the basic noise model, four threshold selection rules (Donoho, 1994) can be implemented with the following threshold return values:

Option	Threshold selection rule	Threshold return
rigrsure	Selection using principle of **S**tein's **U**nbiased **R**isk **E**stimate (SURE)	2.0735
sqtwolog	Fixed form threshold equal to sqrt (2* log (length(s)))	3.7169
heursure	Selection using a mixture of first two options	3.7169
minimaxi	Selection using minimaxi principle	2.2163

Step 4: Reconstruction:
Reconstruct the signal using the approximation coefficients of level M and the modified detail coefficients of levels from 1 to M. Reconstruction is nothing but the up-sampling of the approximation and the modified detailed coefficients to M levels and adding them.

3. Atmospheric wavelets

After applying the above algorithm for a number of data, it was found that a set of wavelets is used more frequently. These wavelets were then analyzed and some of their properties were changed, and a new set of wavelets was designed for their better performance with radar returns. The new set of wavelets was called *Atmoslets* named after the field of signal processing they are used.

The frequently used wavelets are Sym3, Sym8, Coif3, Coif1, Db1 and Db4. The wavelet functions of these wavelets can be obtained by up-sampling the high-pass filter coefficients of the respective Wavelet and convolving it with the low-pass filter coefficients. By performing a number of such iterations, the wavelet function of the respective wavelet can be modified to the near perfect form. As the noise is high-frequency signal, the wavelet function must be as nearest to the high-frequency shape as possible. Because of its nearest shape, the value of correlation is high and noise can be removed using thresholding.

By performing such iterations, the wavelet functions obtained are used in the above algorithm and found improvement in SNR when compared to the normal wavelets. So, a set of six such wavelets are obtained from the respective wavelets with notation 'atm 1 to atm 6'. In general, these wavelets are called *Atmospheric Wavelets*. The wavelet functions of the atmoslets are shown in the Figure 2.

The following are filter coefficients of the atmospheric wavelets, which satisfy the criteria for wavelet filter coefficients

Atm1: *[0.0013 −0.0002 −0.0106 0.0027 0.0347 −0.0192 −0.036 0.2577 0.5496 0.3404 −0.0433 −0.1013 0.0054 0.0224 −0.00047 −0.0024]*
Atm2: *[0.2352 0.5706 0.3252 −0.0955 −0.0604 0.0024]*
Atm3: *[0.0013 −0.0002 −0.0106 0.0027 0.0347 −0.0192 −0.0367 0.2577 0.5496 0.3404 −0.0433 −0.1013 0.0054 0.0224 −0.0004 −0.0024]*
Atm4: *[−0.0514 0.2389 0.6029 0.2721 −0.0514 −0.0111]*
Atm5: *[−0.0027 0.0055 0.0166 −0.0465 −0.0432 0.2865 0.5613 0.3030 0.0508 −0.0582 0.0244 0.0112 −0.0064 −0.0018 0.0008 0.0003 −0.0001 0.0000]*
Atm6: *[0.0006 −0.0012 −0.0052 0.0114 0.0189 −0.0575 −0.0397 0.2937 0.5531 0.3072 −0.0471 −0.0680 0.0278 0.0177 −0.0108 −0.0040 0.0027 0.0009 −0.0004 −0.0002 0.0000 0.0000 −0.0000 −0.0000]*

The following are some of the observations made after the denoising has been performed using atmospheric wavelets. Figure 1(a) is power spectrum of radar returns without denoising; Figure 1(b) depicts the Doppler height profile obtained without denoising. Figure 3(a) is the power spectrum of radar returns after denoising; Figure 3(b) depicts the Doppler height profile obtained after denoising.

- Figure 3(a) clearly shows the removal of high-frequency noise components from the power spectrum shown in Figure 1(a), and increases SNR. Increase in SNR helps in the detection of Doppler height profile even at greater heights in the midst of noise as it is demonstrated in Figure 3(b).

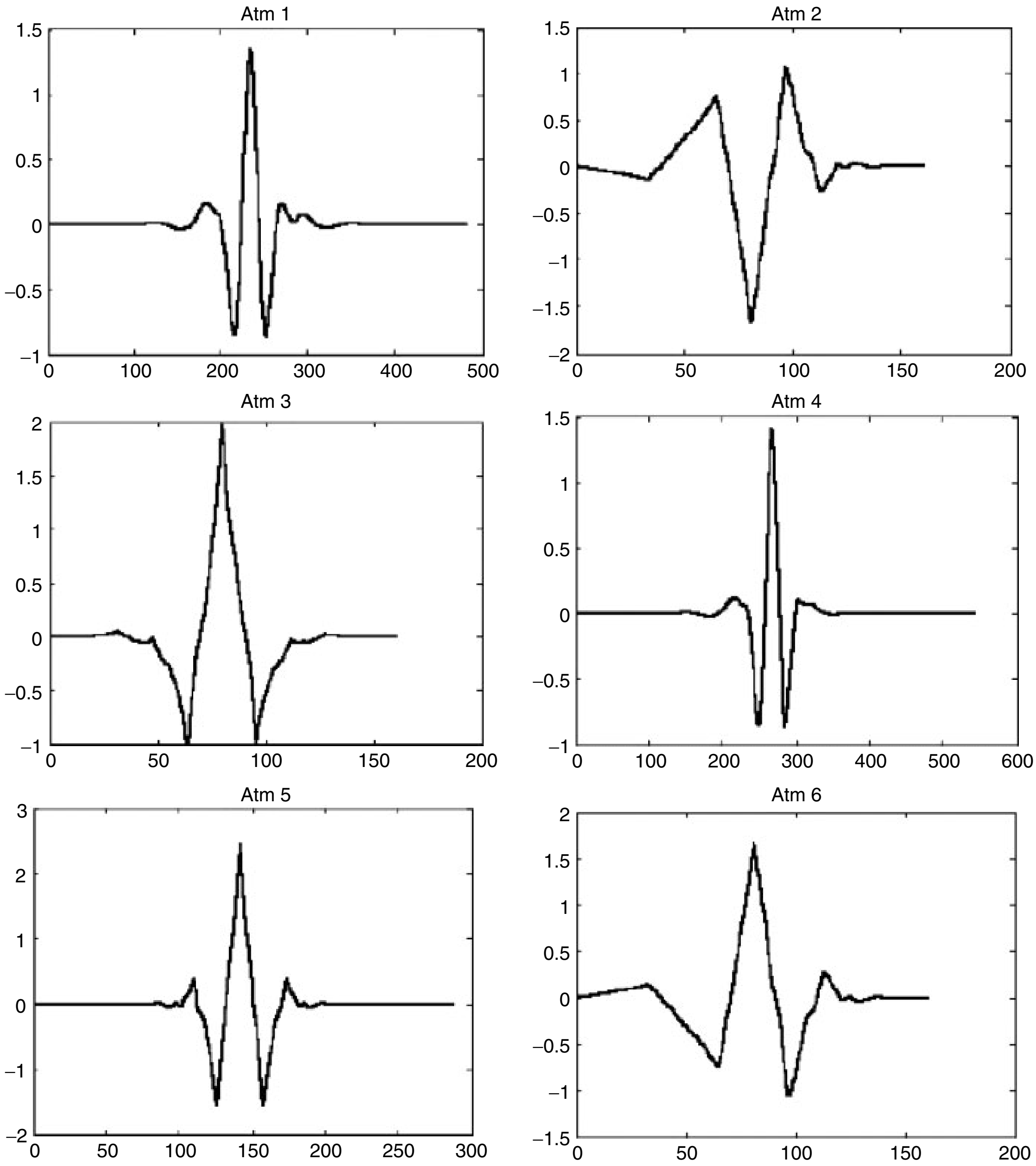

Figure 2. Wavelet functions of the atmoslets

- It is observed that as level of the decomposition increases, the probability of filtering out of the genuine peak also increases; hence, lower-level wavelets should be used for denoising atmospheric signals.
- Another point that is noteworthy is that orthogonal wavelets provide better denoising when compared to other wavelets especially for signals predominant with Gaussian noise.

The improvement in SNR achieved by denoising using the atmospheric wavelets is demonstrated in Figure 4. The plot shows the SNRs obtained at different heights with and without denoising for the noisy data. The plot clearly shows the descent in the SNR plot without denoising at higher altitudes. The following are some of the observations that can be made on the plot.

- Owing to denoising, there is an improvement of SNR by 10 to 15 dB.
- Because of improved SNR, it is possible to detect the echoes and thus predict and detect the wind profile more accurately even under noisy conditions, because detection is directly related to SNR of the signal.

The power spectrum obtained in the presence of strong interference is shown in Figure 5(a). It can be seen that the interference dominated the signal from the height of 12 km to the end of the frame. Hence, the Doppler height profile could be detected up to 15 km as shown in Figure 5(b). The same data is denoised using the atmospheric wavelet and the resultant power spectrum is shown in Figure 6(a), which shows clearly the removal of the interference. Figure 6(b) shows the

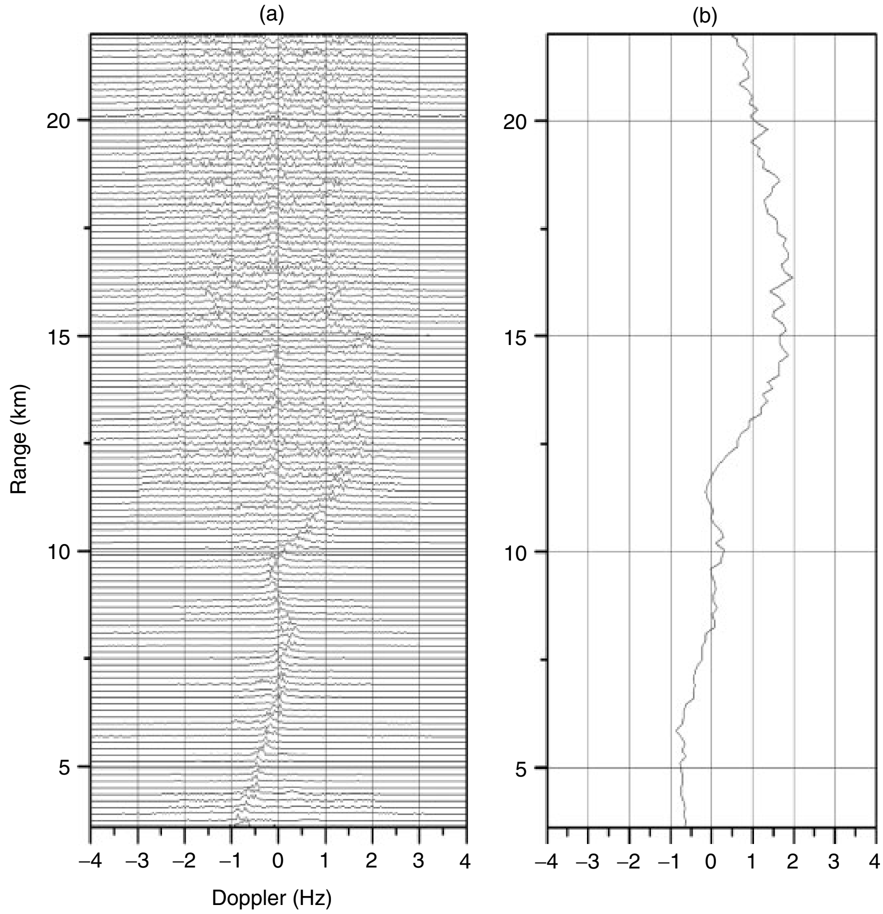

Figure 3. (a) Doppler power spectrum after denoising with atmospheric wavelet (b) Doppler height profile extracted from the spectrum after denoising

increase in the Doppler height profile obtained because of denoising.

4. Wind profile detection using parametric method (harmonic decomposition)

Wind profile detection based on harmonic decomposition (Parametric Method) (Proakis and Manolakis) relies on the eigen analysis of the autocorrelation function of the signal. The autocorrelation is the spectral characteristic of the signal in time domain. The autocorrelation of the time domain data is obtained for different lags. The eigenvalues of the matrix formed by the autocorrelation coefficients are computed. The corresponding Eigenvectors are also found out. The minimum of the eigenvalues, which corresponds to the signal, is found and the corresponding eigenvector is solved for the roots. The roots correspond to the frequency of the signal data. The algorithm using parametric method based on harmonic decomposition can be stated as follows.

Pisarenko harmonic decomposition:

Pisarenko harmonic decomposition method is an Eigen analysis algorithm, which is used in power spectrum estimation of randomly phased sinusoids corrupted with white Gaussian noise. The algorithm is based on an Eigen decomposition of the correlation matrix of the noise-corrupted signal.

This Pisarenko method is based on the use of a noise subspace eigenvector to estimate the frequencies of the sinusoids.

For p randomly phased sinusoids in additive white noise, the autocorrelation values are

$$\gamma_{yy}(0) = \sigma_{\mathrm{w}}{}^2 + \Sigma P_i;$$
$$\gamma_{yy}(k) = \Sigma P_i \cos 2\pi fik \qquad k! = 0 \tag{1}$$

where $P_i = \mathrm{A}_i{}^2/2$ is the average power in the ith sinusoid and A_i is the corresponding amplitude. In matrix form, it can be written as:

$$\begin{pmatrix} \cos 2\pi f_1 & \cos 2\pi f_2 & \dots\dots\dots \cos 2\pi f_p \\ \cos 4\pi f_1 & \cos 4\pi f_2 & \dots\dots\dots \cos 4\pi f_p \\ . & . & . \quad . \\ . & . & . \quad . \\ \cos 2\pi pf_1 & \cos 2\pi pf_2 & \dots\dots\dots \cos 2\pi pf_p \end{pmatrix} \times \begin{pmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ P_p \end{pmatrix} = \begin{pmatrix} \gamma_{yy}\ (1) \\ \gamma_{yy}\ (2) \\ \gamma_{yy}\ (p) \end{pmatrix} \tag{2}$$

If the frequencies f_i, $1 < \mathrm{I} < p$ are known, this equation may be used to determine the powers of the

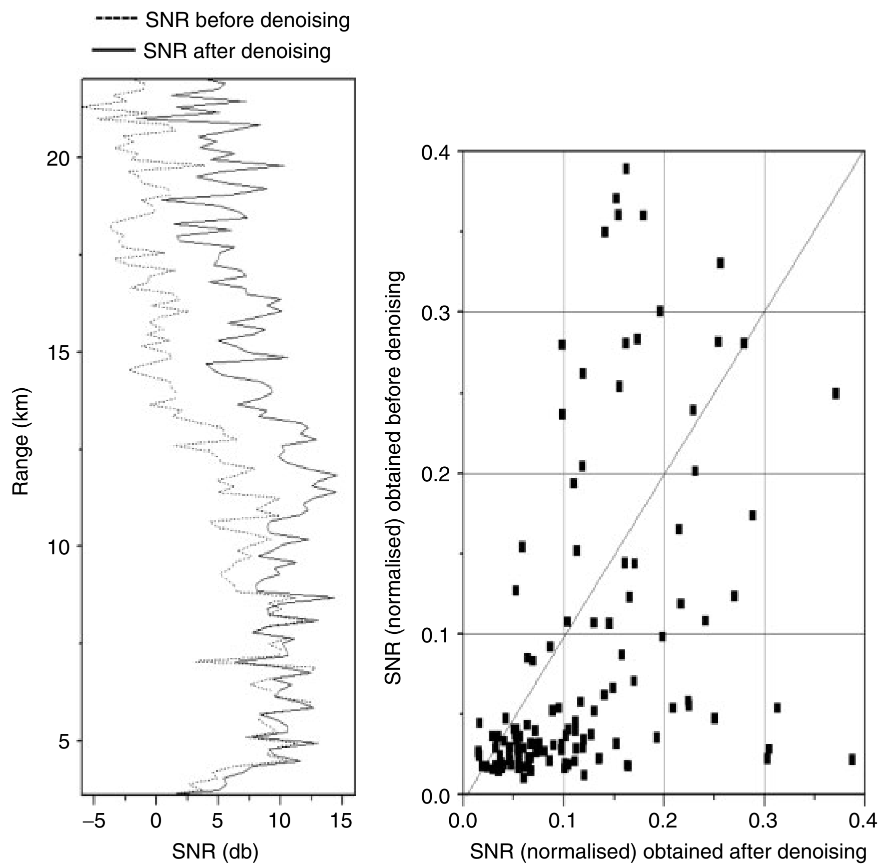

Figure 4. (a) Height profiles of signal-to-noise ratio (SNR) estimated before and after denoising (b) Scatter plot between SNR's before and after denoising

sinusoids. The estimates r_{xx} (m) are used in place of γ_{xx} (m). By using the powers, the noise variance can be obtained from Equation (1) as

$$\sigma_w{}^2 = r_{yy}(0) - \Sigma P_i$$

The problem that remains is to determine the p frequencies f_i, $1 < I < p$, which, in turn, require knowledge of the eigenvector **a** corresponding to the eigenvalue $\sigma_w{}^2$. Pisarenko observed that for an auto regressive moving average (ARMA) process consisting of p sinusoids in additive white noise, the variance $\sigma_w{}^2$ corresponds to the eigenvalue of Γ_{yy}, when the dimension of the autocorrelation matrix equals or exceeds $(2p+1) \times (2p+1)$. The desired ARMA coefficient vector corresponds to the eigenvector associated with the minimum eigenvalue. Therefore, the frequencies f_i, $1 < I < p$ are obtained from the roots of the polynomial in (2), where the coefficients are the elements of the eigenvector a corresponding to the minimum eigenvalue $\sigma_w{}^2$.

The Pisarenko harmonic decomposition method can be put in a nutshell as follows. First, the autocorrelation matrix R_{yy} is estimated from the data. Then, the minimum eigenvalue and the corresponding eigenvector are found out for the autocorrelation matrix. The minimum eigenvector yields the parameters of the ARMA $(2p, 2p)$ model. By using the Equation (1), the roots corresponding to the frequencies $\{f_i\}$ are computed. By using these frequencies, the Equation (1) is solved for the signal powers $\{P_i\}$ by substituting the estimates r_{yy} (m) for γ_{yy} (m).

The algorithm can be explained in the following steps.

- The autocorrelation function of the time domain data is computed and the correlation matrix is formed using the correlation coefficients. The matrix must be square and the lower triangular matrix must be conjugate of the upper triangular matrix.
- The eigenvalues and the eigenvectors of the correlation matrix are computed.
- The minimum eigenvalue and the minimum eigenvector are found out from the Plot. This minimum eigenvector corresponds to the signal and the remaining corresponds to the noise.
- The eigenvectors are solved for the roots. The roots correspond to the normalized frequency of the signal.

Figure 7 shows the profile drawn using the harmonic decomposition algorithm for the data whose power spectrum is shown in Figure 1. The following are some of the observations made on profile drawn using the harmonic decomposition algorithm.

- The harmonic decomposition algorithm uses the quadrature and in-phase components of the radar returns when compared to other algorithms, which use power spectrum data.

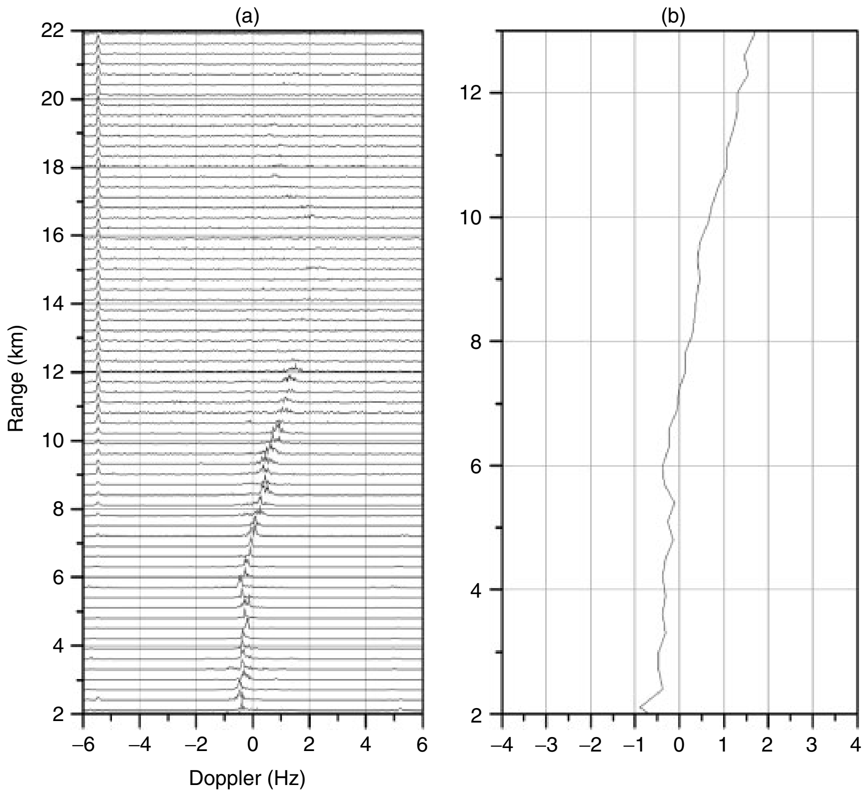

Figure 5. (a) Doppler power spectrum corrupted with interference (b) Doppler height profile obtained in the presence of interference

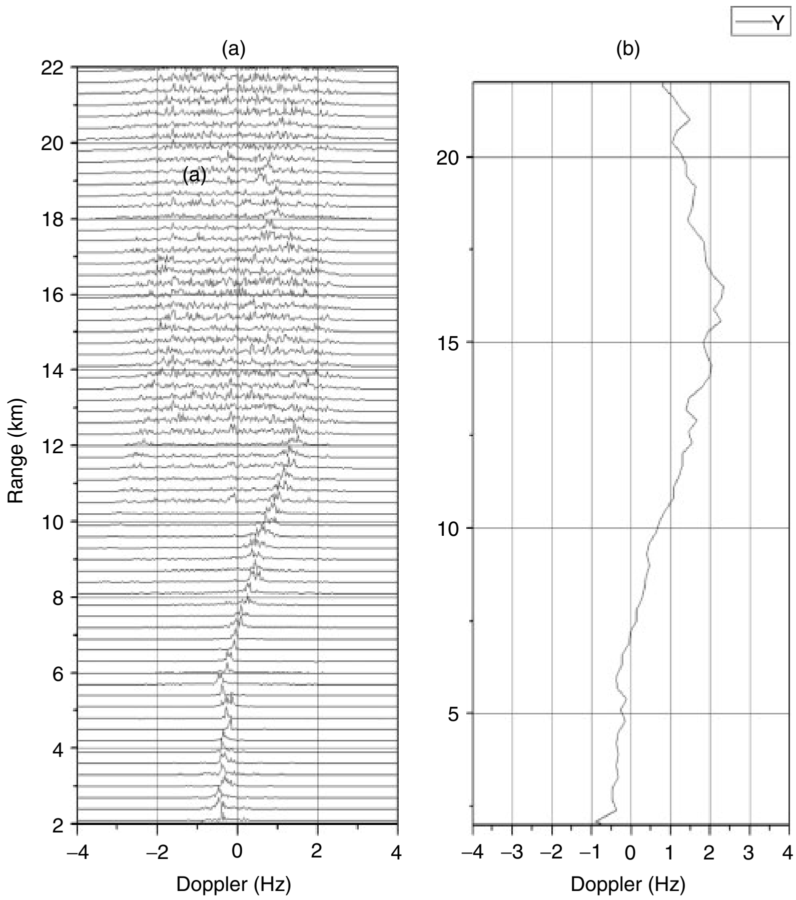

Figure 6. (a) Doppler power spectrum after denoising with atmospheric wavelet (b) Doppler height profile obtained after removal of interference

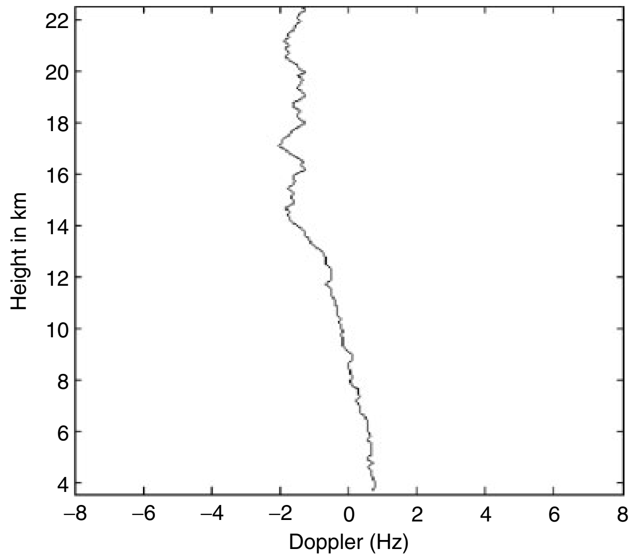

Figure 7. Profile plot drawn using the Dopplers provided by the harmonic decomposition algorithm

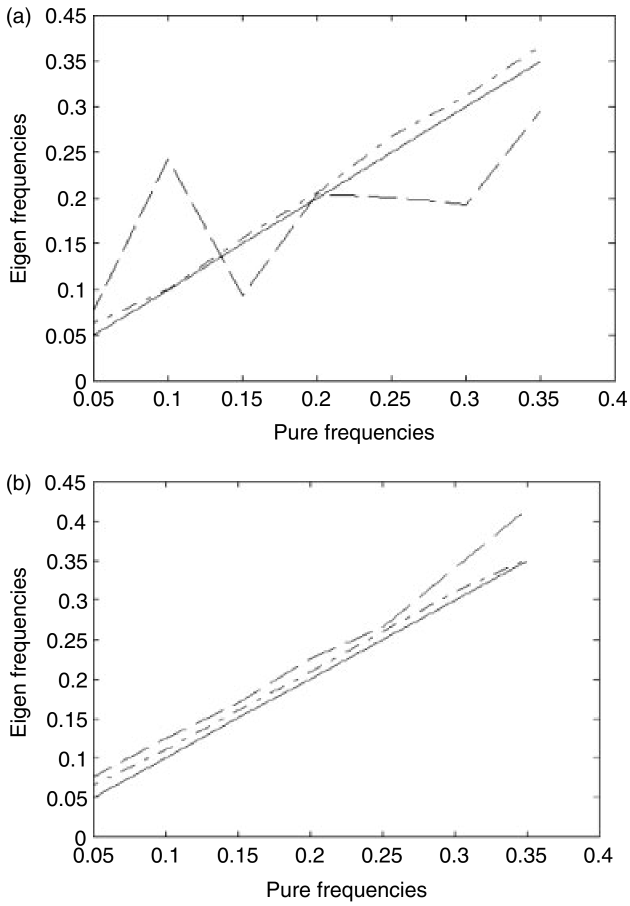

Figure 8. (a) Pure frequencies versus eigen frequencies. Solid, pure; dash, noisy; dash-dot, denoised SNR = −10 dB (b). Pure frequencies versus eigen frequencies. Solid, pure; dash, noisy; dash-dot, denoised (b) SNR = −5 dB

- The algorithm estimates Doppler frequencies consistently up to a height of 20 km even under very low SNRS.
- The algorithm produces slight errors in Doppler frequencies at the altitudes of about 20 to 22 km where the signal has very low SNRs.

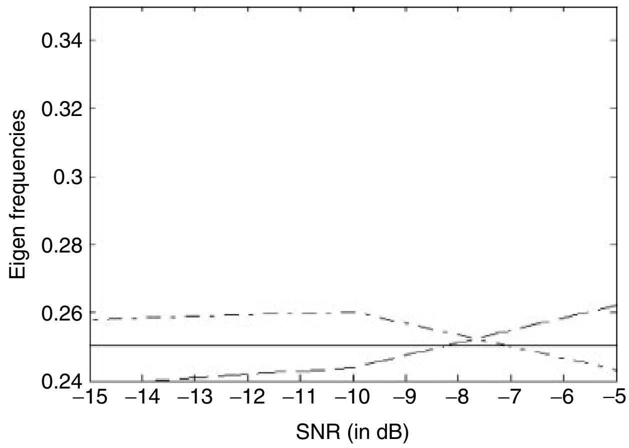

Figure 9. SNR versus eigen Frequencies. Solid, pure; dash, noisy; dash-dot, denoised for pure frequency 0.25 Hz

In order to overcome the limitations of the harmonic decomposition algorithm in processing the atmospheric radar returns, both the above algorithms are combined into one hybrid algorithm. The hybrid algorithm has been discussed in the following section.

5. Wind profile algorithm using both parametric and non-parametric methods (hybrid algorithm)

An algorithm, which is a hybrid of the non-parametric method and the parametric method, has been developed. The algorithm uses the non-parametric method for spectrum cleaning and parametric method for estimating the power spectrum. In spectral cleaning, wavelet decomposition is used. In power spectrum estimation, harmonic decomposition is used. In spectrum cleaning, the algorithm discussed in the denoising section is used.

The cleaned data is then sent to the harmonic decomposition block where the Doppler frequencies are estimated using the algorithm presented in the previous section.

The hybrid algorithm is divided into three parts as given below.

- Spectrum cleaning
- Harmonic decomposition or eigen analysis
- Profile plotting algorithm.

5.1. Validation of the algorithm

The algorithm has been tested on the simulated signals for different SNRs and frequencies, and the following are some of the observations that are worthwhile.

- Consistently estimating the frequencies.
- For very low SNRs, Harmonic decomposition after denoising has improved the efficiency of spectral estimation.

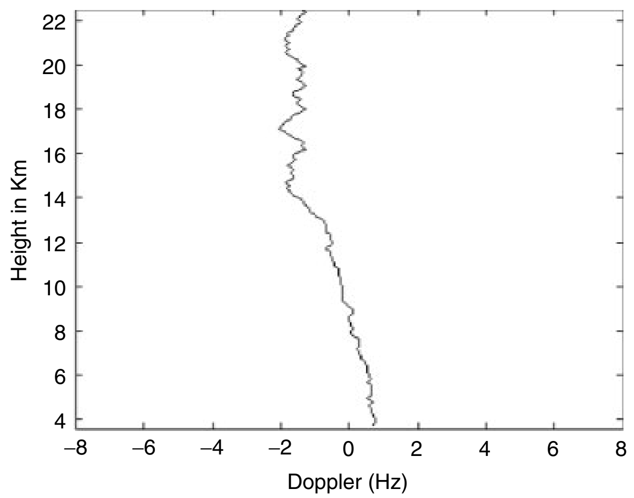

Figure 10. Profile plot drawn using the Dopplers provided by hybrid algorithm, which uses wavelets for spectral cleaning and harmonic decomposition for Doppler estimation

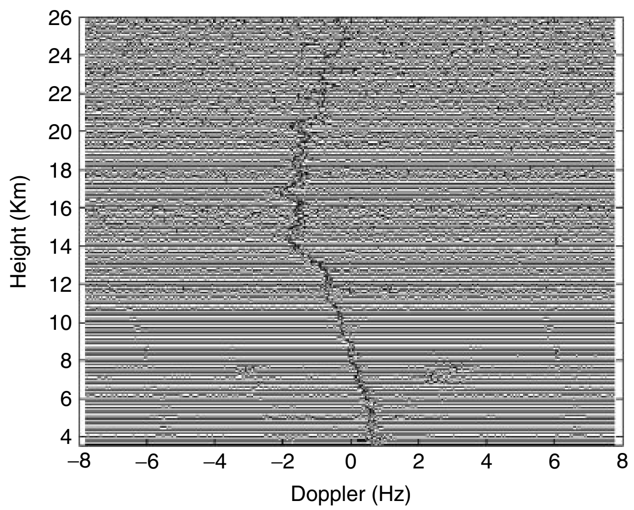

Figure 11. Comparison of profiles drawn using the hybrid algorithm and harmonic decomposition algorithm individually on the power spectrum plot of the WEST beam blue profile → hybrid algorithm red profile → harmonic decomposition algorithm

Figure 8 shows the plot drawn between the eigen frequencies and real frequencies for different SNRs (−10 dB and −5 dB). The solid line corresponds to the pure frequencies, dash line corresponds to the noisy data and the dash-dot line corresponds to the denoised frequencies.

Figure 9 shows the plot drawn between SNR and eigen frequencies for different frequencies. The solid line corresponds to the pure frequencies, dash line corresponds to the noisy frequencies and dash-dot line corresponds to the denoised frequencies. After obtaining encouraging results for the simulated signals, the proposed hybrid algorithm is tested for the atmospheric data and the following are some of the results obtained using our algorithm.

Figure 10 shows the profile drawn using the proposed hybrid algorithm that uses non-parametric wavelet denoising for spectral cleaning and parametric harmonic decomposition for Doppler estimation. Figure 11 gives a comparison between hybrid algorithm and harmonic decomposition. The blue profile corresponds to hybrid algorithm and the red one corresponds to the harmonic decomposition.

6. Conclusions

From the results shown above, the following conclusions can be made.

- The blue profile corresponding to hybrid algorithm is smoother than the one corresponding to the harmonic decomposition. Hence, the blend of non-parametric and parametric methods yields better results compared to either harmonic decomposition or wavelets.
- The hybrid algorithm also computes accurate frequencies for high noisy data. The combination makes the algorithm more efficient in very low SNR regions.

Data observation table:

Period of observation	***2001–2003***
Pulse width	*16 μs*
Range resolution	*150 m*
Inter pulse period	*1000 μs*
No of beams	*6 (E10y, W10y, Zy, Zx, N10x, S10x)*
No of FFT points	*512*
No of incoherent integrations	*1*
Maximum Doppler frequency	*3.9 Hz*
Maximum Doppler velocity	*10.94 m/s*
Frequency resolution	*0.061 Hz*
Velocity resolution	*0.176 m/s*

- $E10y$ = *east–west polarization with offzenith angle of* 10°
- $W10y$ = *east–west polarization with offzenith angle of* 10°
- $N10x$ = *north–south polarization with off zenith angle of* 10°
- $S10x$ = *north–south polarization with off zenith angle of* 10°

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